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An Examination of Mathematical Knowledge for Teaching in Professional Development Settings

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AN EXAMINATION OF THE DOMAINS OF MATHEMATICAL KNOWLEDGE FOR TEACHING IN PROFESSIONAL DEVELOPMENT SETTINGS

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ABSTRACT

Researchers have laid the foundation for what mathematics teachers need to know, a construct referred to as Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008). There is, however, a lack of robust research regarding how Mathematical Knowledge for Teaching (MKT) is developed within professional development settings.

The purpose of this qualitative research study, therefore, was to examine how different foci of professional development revealed different domains of MKT. In the immersion setting, the professional development focused on engaging teachers in discourse surrounding teachers' completion of mathematical tasks. In the practice-based setting, the professional development focused on engaging teachers in discourse surrounding student work associated with the mathematical tasks.

To capture the domains of MKT in the two professional development settings, the researcher collected data from two groups of participants within two different schools. Each group of participants attended four professional development sessions in which video recorded professional development sessions, participant weekly reflections, and observation guides captured emerging MKT domains. To answer the research questions, the researcher utilized qualitative means for analyzing data taken from each of the data sources. Transcriptions from the recorded sessions and the participant weekly reflections provided the researcher with the bulk of her findings.

In reviewing the domains represented from the immersion setting data, the domain that appeared more often than the rest was knowledge of content and students. In reviewing the

domains represented from the practice-based professional development data, the domain that appeared more often than the rest was specialized content knowledge. At times, other MKT domains were present in participants' conversations and written work; however, knowledge of content and students and specialized content knowledge were the most prevalent.

Two findings stemmed from this study. One, identifying the domains of MKT is difficult when the ideas of the participants are flawed both mathematically and pedagogically. Two, different professional development sessions lead participants to focus on different domains of MKT. To this end, the goal of professional development should drive the type of professional development setting employed when developing teachers' MKT.

DEDICATION

This work is dedicated to elementary education teachers who struggle to make a difference in the lives of children every day. Thank you for your dedication to the field of teaching and learning.

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I would like to thank Dr. Angela Barlow for her mentorship, collegiality, and friendship. Without her support this work would not have been possible. I would also like to thank my committee members, namely Dr. Doug Davis, Dr. Sarah Blackwell, and Dr. Carol Livingston, for their support and guidance. In addition, I would like to thank the outstanding group of women who I have the privilege to call my very best friends. Their listening ears, words of encouragement, shoulders to cry on, and often spare bedrooms to sleep in, have pushed me to achieve this honor. Finally, I would like to thank my Lord and Savior, Jesus Christ, through whom all things are possible.

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CHAPTER I: INTRODUCTION

Introduction

Effective mathematics teaching is more than knowing how to work mathematics problems in front of students (Hill, Sleep, Lewis, & Ball, 2007). It requires knowing more mathematics than what one is teaching and how to translate that knowledge to the students. Effective teaching, however, is a difficult task because many teachers do not have the necessary fundamental mathematics knowledge to support their pedagogical knowledge (Sowder, 2007). According to Sowder, “If mathematics learning is to be useful in the classroom, it cannot be separated from the learning of pedagogical knowledge” (p. 169). Additionally, Sowder recognized that sufficient mathematics knowledge requires the appropriate pedagogical knowledge to teach mathematics. Acquiring both leads teachers to develop self-efficacy as mathematics teachers (Sowder, 2007). To obtain both mathematical content knowledge and its supporting pedagogical knowledge, teachers must look to professional development programs. Such programs should provide teachers with inventive ideas about the teaching and learning of mathematics.

Purpose of Study

The purpose of this qualitative research study was to examine the type of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008) that emerged in two professional development settings. In one school setting, the professional development focused on engaging teachers in discourse surrounding teachers’ completion of mathematical tasks. In the second school setting, the professional development focused on engaging teachers in discourse surrounding student work associated with the mathematical tasks. The type of mathematical knowledge that emerged in those two different settings was the focal point of this study. Because this study's focal point was Mathematical Knowledge for Teaching and professional development as a means for developing this knowledge, the following paragraphs will provide a brief description of each.

Professional Development

To gain the knowledge necessary to establish classrooms that align with national standards (National Council of Teachers of Mathematics, 2000; Common Core State Standards Initiative, 2010) and state standards (e.g. Mississippi Department of Education, 2007), teachers must experience transformative learning as opposed to additive learning (Smith, 2001). That is to say, teachers must experience professional development that causes them to rethink completely their views of mathematics and its teaching, thus transforming how they view mathematics instruction. This is in contrast to professional development that engages teachers in additive learning where information is given to the teachers with the expectation that it be added to an already existing knowledge base. According to Smith (2001), professional development that engages teachers in transformative learning must address changes related to "what it means to know and understand mathematics, the kinds of tasks in which their students should be engaged, and, finally, their own role in the classroom" (p. 4).

In designing transformative professional development that addresses teachers' Mathematical Knowledge for Teaching, mathematics educators assert that professional development must engage teachers in collaboratively completing tasks that enable them to expand their views of what it means to know and understand mathematics (National Research Council, 2001). These tasks should align with the mathematics that teachers are required to teach

(Cohen, 2004) based on state frameworks. When completion of such tasks includes consideration of student thinking, the result is an increase in student achievement (Cohen & Hill, 2001). In addition, teachers should engage in tasks that facilitate their understanding of the development of the mathematics in earlier grade levels through later grade levels (Ball, Thames, & Phelps, 2008; Cohen, 2004).

While key components of transformative professional development have been identified, the research base supporting these components has only begun to be established. This research study, therefore, examined two particular components, namely completion of mathematical tasks and consideration of student thinking. Specifically, these components were examined according to the domains of Mathematical Knowledge for Teaching that emerged. These domains will be described in the section that follows.

Mathematical Knowledge for Teaching

Ball et al. (2008) defined the construct of Mathematical Knowledge for Teaching (MKT) as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). To communicate the types of mathematical knowledge involved in teaching, they divided MKT into six domains that collectively represent the mathematical knowledge needed for teaching mathematics. This framework provides mathematics educators with a lens for examining the mathematical knowledge exhibited by teachers in professional development sessions. Table 1 provides a brief description and mathematical example of the six MKT domains. A description with supporting research is provided in the next chapter.

Table 1

The Six Domains of Mathematical Knowledge for Teaching

Domain	Description	Example
Common Content Knowledge (CCK)	Mathematical knowledge that is viewed as common to all persons, regardless of their daily work	Adding fractions
Specialized Content Knowledge (SCK)	Mathematical knowledge that is unique to teachers	Representing the addition of fractions with a picture or model
Horizon Content Knowledge (HCK)	Knowledge of the mathematics that lies ahead, on the horizon, related to the topic	Recognizing, for example, the link between fraction operations and rational expressions
Knowledge of Content and Students (KCS)	Knowledge of how students come to understand mathematics	Selecting appropriate tasks for students to engage, utilizing the student work generated from the task to identify students' understandings and misunderstandings, and finally making instructional decisions based on these findings
Knowledge of Content and Teaching (KCT)	Knowledge of how to develop mathematical understanding in students	
Knowledge of Content and Curriculum (KCC)	Knowledge of how the curriculum facilitates the development of students' mathematical understandings	

The development of MKT as a construct began in 1986 when Shulman attempted to bridge the gap between content knowledge and teaching practices. It is the work of Ball et al. (2008), however, that described the knowledge teachers need to carry out the work of teaching mathematics. While a vast amount of literature supporting MKT is available, to date, the way in which teachers acquire MKT has yet to be determined (Ball et al., 2009).

Significance of the Study

Professional development programs for teachers hold the potential for transforming the way in which mathematics is taught. The methods teachers employ in teaching mathematics are impacted by their beliefs (Phillipp et al., 2007; Thompson, 1992). In addition, there is agreement within the mathematics education community that teachers must have a deep understanding of the mathematics they teach and yet they often do not (Ma, 1999). Research has documented that professional development should be sustained and rooted in practice (Ball et al., 2008). The significance of this study, therefore, was in its examination of how different foci of professional development revealed different domains of Mathematical Knowledge for Teaching.

Research Questions

In examining the role of Mathematical Knowledge for Teaching in professional development, the following research questions were posed.

1. What domains of Mathematical Knowledge for Teaching emerge in a professional development setting centered on the completion of mathematical tasks?
2. What domains of Mathematical Knowledge for Teaching emerge in a professional development centered on the analysis of student work?

Summary

Students' mathematical successes are directly linked to the mathematical knowledge that teachers possess (Kazemi et al., 2009). How this knowledge is developed within teachers, however, is not clear. This study examined how two different professional development settings facilitated teachers' discussions involving Mathematical Knowledge for Teaching. The results of this study lend themselves toward the establishment of the effectiveness of two components of transformative professional development. Specifically, the results hold the potential for demonstrating how different forms of professional development support teachers in reflecting on different components of mathematics teaching. In the next chapter a review of related literature will be shared that supports the significance of this study.

CHAPTER II: REVIEW OF LITERATURE

Introduction

Researchers have laid the foundation for what mathematics teachers need to know, a construct referred to as Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008). There is, however, a lack of robust research regarding how Mathematical Knowledge for Teaching is developed within professional development settings. The focus of this literature review, therefore, will be on Mathematical Knowledge for Teaching and professional development as a means for developing this knowledge.

This chapter will begin with a description of pedagogical content knowledge. Next, the development of Mathematical Knowledge for Teaching and its six domains is described. Finally, current professional development program designs and implementation methods are presented.

Pedagogical Content Knowledge

In 1986, Shulman and his colleagues introduced the construct, pedagogical content knowledge (PCK). This construct was evolutionary for the education field due to its emphasis on the intersection between content knowledge and teaching practices. Additionally, this construct was considered complimentary to general pedagogical knowledge while also identifying the importance of knowledge about subject matter. Specifically, PCK included teachers' knowledge of the ideas students find interesting or difficult, the most useful representations for teaching an idea, and students' typical understandings and misunderstandings (Hill, Schilling, & Ball, 2004).

Although this work was crucial and emphasized the dimensions of teacher knowledge, it was not, however, specific to the field of mathematics. Ma's (1999) work, however, was central to knowing and teaching elementary mathematics. Ma compared Chinese and American elementary teachers who were engaged in discussing and completing mathematical tasks. This comparison exposed major differences in how the two groups of teachers talked about their work and reflected on the processes in which they engaged. As a result of this work, Ma introduced the term Profound Understanding of Fundamental Mathematics (PUFM) that she described as "an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough. Although the term profound is often considered to mean intellectual depth, its three connotations, deep, vast, and thorough, are interconnected" (Ma, 1999, p. 120).

Shulman's PCK and Ma's PUFM constructs prompted mathematics educators to reconsider what type of knowledge about the teaching and learning of mathematics are needed by teachers. To this end, a new research construct emerged. This construct, Mathematical Knowledge for Teaching, will be defined and discussed in the following section.

Mathematical Knowledge for Teaching

What teachers need to know and be able to do to teach mathematics effectively has been widely analyzed (Ball, Hill, & Bass, 2005; Ma, 1999; NCTM, 2000). In 2008, Ball, Thames, and Phelps introduced the construct Mathematical Knowledge for Teaching (MKT) and described it as the knowledge needed to carry out the work of teaching mathematics. Ball and colleagues emphasized the fact that this description begins with teaching and not the teacher. From their work, six domains of MKT emerged: common content knowledge (CCK), horizon content knowledge (HCK), specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Collectively, these six domains represent the mathematical knowledge needed for teaching mathematics. One means for mathematics teachers to develop this type of knowledge is through professional development. Recognizing the role of professional development, Suzuka et al. (2009) gave recommendations to professional development leaders who work to enhance teachers' MKT in professional development settings. These recommendations included the following: leaders should engage teachers in metacognition; leaders should have teachers routinely express their thoughts and share their solution processes; and leaders should be explicit about how the tasks in which the teachers are engaged are connected to the work of teaching mathematics (Suzuka et al., 2009).

In consideration of these recommendations, a description of each MKT domain, along with insights to the potential professional development settings hold for developing this type of knowledge will be provided in the following paragraphs.

Common Content Knowledge

Common content knowledge (CCK) is the knowledge of mathematics that is not unique to teaching. This type of knowledge is known and used by others within and outside the mathematics community. For example, the skill of adding fractions is viewed as common content knowledge because it is a skill that everyone should possess, regardless of his or her profession. Research addressing the development of CCK in teachers is not available given that within the mathematics community assumptions are made that teachers already possess CCK. Therefore, there is a more demanding need to develop the other MKT domains within professional development settings (Hill, Schilling, & Ball, 2004).

Specialized Content Knowledge

Specialized content knowledge (SCK) is the knowledge of mathematics unique to teaching mathematics. It requires knowledge of mathematics on a deeper level than just what is being taught to students. For example, being able to represent the addition of fractions with a picture or model represents a mathematical knowledge piece that is unique to teaching mathematics. Teachers must possess knowledge of such models as they plan to teach for conceptual understanding. Other professions do not need this type of mathematical knowledge in their everyday work (Ball et al., 2008). Unlike CCK, researchers have evidence to support the development of SCK in professional development settings. In the following paragraphs, results from research studies that involved this type of knowledge will be shared.

Kazemi, Elliot, Lessig, Mumme, Carroll, and Kelly-Peterson (2009) investigated professional development that was used to build teachers' SCK. The purpose of this study, however, was to offer insight gained by working with professional development leaders. The focus of the work with the professional development leaders was two-fold. First, the leaders recognized the importance of articulating the difference between the mathematics done in professional development and that done in the K-12 classroom. Second, the leaders identified the need to link sociomathematical norms for explanation and the practices for orchestration of classroom discussions to the purposeful development of SCK. Project participants and data sources included 36 leaders from two areas with various levels of experience with conducting professional development. Data was collected from transcripts, questionnaires, and work samples. Two significant themes emerged from this study. First, doing mathematics in professional development is different from doing mathematics in the classroom. This is due largely in part to the fact that teachers hold mathematical knowledge differently than students, i.e., teachers already possess CCK, and in some instances, SCK. Second, the professional development leaders' relationships with teachers are different than the teacher/student relationships. This component to professional development adds a layer of complexity which professional development leaders must recognize when designing programs (Kazemi et al., 2009).

As professional development leaders seek to develop teachers' SCK, the need for instruments that measure this MKT domain becomes apparent. In 2004, Hill and colleagues began to write and pilot such assessment items. These items were early attempts to gauge Mathematical Knowledge for Teaching of elementary mathematics (Hill et al., 2004). Piloting these items in a professional development setting provided evidence that CCK and SCK were related yet were not completely equivalent. These results suggested that the possibility existed that teachers might have well-developed CCK yet lack the specific kinds of knowledge that are unique to teaching mathematics or SCK. Additionally, this indicated that teachers could develop SCK from professional development yet still lack knowledge of mathematical content (Hill et al., 2004).

Recognizing the uniqueness and importance of SCK, Hill and Ball (2004) found that there were no measurement items available to gauge teachers' SCK accurately. To this end, Hill and colleagues created assessment items based on elementary number concepts and operations to test teachers' Mathematical Knowledge for Teaching. These assessment items were piloted at various summer institutes in California for in-service teachers. The baseline knowledge of the participants and different instructional methods used within the professional development settings, however, limited the results and interpretations of the research. Despite those limitations, the results revealed that teachers could learn mathematics for elementary school teaching in the context of a single professional development program (Hill & Ball, 2004).

Although much work in creating assessment items to measure teachers' SCK has been conducted, Hill, Sleep, Lewis, and Ball (2007) have noted that there is still a need for improvement in this area. Examining how SCK emerges in professional development settings through qualitative means may be the key to those improvements.

Horizon Content Knowledge

Horizon content knowledge (HCK) is the knowledge of what students need to know for future mathematics work. Additionally, this type of knowledge includes understanding students' prior knowledge. As an example, when considering addition of fractions, teachers should know the mathematics that lies ahead, on the horizon, related to the topic. In this case, a teacher who is teaching students about addition of fractions should recognize not only that other operations such as multiplication will be taught but also that in future grade levels fraction operations will be linked to operations with rational expressions in their algebra classes. Such horizon knowledge supports the teacher's decision processes in the development of students' understandings (Ball et al., 2008). Currently, there is a lack of research that specifically addresses the development of teachers' HCK.

Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum

The remaining three domains of MKT collectively represent pedagogical content knowledge. As described by Shulman (1986), PCK is the intersection between content knowledge and teaching practices. In developing the MKT construct, Ball and colleagues (2008) divided PCK into three domains, namely knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). These domains reveal that MKT includes knowing how students come to understand mathematics, how to develop this understanding in students, and how the curriculum facilitates that process. Related to addition of fractions, the teacher must select appropriate tasks for students to engage, utilize the student work generated from the task to identify students' understandings and misunderstandings, and finally make instructional decisions based on these findings. These processes utilize all three of the domains constituting pedagogical content knowledge (Ball et al., 2008).

Developing teachers' PCK initially focused on moving teachers beyond show-and-tell methods of teaching (Stein, Engle, Smith, & Hughes, 2008). To this end, Stein and colleagues (2008) presented five practices for helping teachers move beyond this method when teaching mathematics. The purpose of their pedagogical model was to specify five practices teachers could learn to use for effective class discussions. These five practices as suggested by the researchers included anticipating students' understandings or misunderstandings, monitoring student progress through problem-solving tasks, purposefully selecting student presentations, purposefully sequencing student presentations, and connecting student responses to the mathematical concept being developed in the lesson (Stein et al., 2008). In order for teachers to engage in these moves successfully they must utilize their MKT, specifically their PCK.

Embedded within these five practices is another PCK component, noticing of students' thinking. Recognizing this component, Jacobs, Lamb, and Philipp (2009) focused on groups of teachers and the extent to which they noticed students' mathematical thinking. Participants in this research study included 131 elementary teachers with three groups of practicing teachers and one group of prospective teachers. Prospective teachers were undergraduates taking their first mathematics content course. Practicing teachers were grouped as initial participants (K-3 teachers who were about to attend professional development for the first time), advancing participants (K-3 teachers in their first two years of professional development training), or emerging participants (K-3 teachers with four years of professional development training and who were taking formal and informal leadership roles at their schools). The measures used for this study were artifacts from practice, which included video clips and students' sample work. Findings from this study demonstrated that teachers who had increased opportunities with students noticed thinking on the professional development tasks more than those with fewer experiences with students. Researchers found that the teachers' expertise grew with experience and continued to grow when they had at least two years of professional development experience. As a result of this discovery, a theme emerged. The researchers identified teachers as either displaying robust, limited, or lack of evidence of professional noticing. Robust evidence was described as those teachers who focused on making sense of strategy details in a variety of ways. Teachers identified as providing limited evidence provided a less in-depth description of student strategies. Furthermore, this group used broad terms when describing their professional noticing while also overgeneralizing their conclusions. The lack of evidence group of teachers lacked focus on the students' thinking and only discussed the teachers' moves (Jacobs et al., 2009). While professional noticing is tremendously important to teaching mathematics, it is the aspect of analyzing student work in this study that is particularly interesting for developing teachers' MKT.

The advantages of analyzing student work samples were explored in a study conducted by Groth and Burgess (2009). In this study, thirty mathematics teachers engaged in online activities surrounding the analysis, and their discussions about classroom artifacts were examined. The researchers found that classroom artifacts, namely student work samples, motivated the teachers to talk about the mathematics content, talk about the students, and talk about the pedagogical content knowledge needed in their instructional practices (Groth & Burgess, 2009).

Similarly, Kazemi and Franke (2003) studied professional development in which teachers were engaged in workgroup meetings where student work samples were the focus. These researchers found that teachers who studied student work samples aimed at developing a way of interpreting the various mathematical strategies students used built a framework for interpreting different student mathematical ideas. As a result of this examination, the researchers noted a need to facilitate the discussion about the student work samples because the teachers were not accustomed to this type of professional development discourse. Therefore, discussions of student work were combined with discussions of mathematics, student reasoning, and instruction. Kazemi and Franke stated the following:

> Launching into analysis of student work provokes teachers to understand the student's particular solution at the same time that it presses them to elaborate the disciplinary knowledge required to make full sense of what the student did. Making sense of student's strategies could be an indirect way for teachers to wrestle with the mathematical ideas themselves. Moreover, these discussions initiated questions about the instructional content in which the work was produced and about pedagogies that could advance the student's thinking. (p. 7)

Mathematical Knowledge for Teaching Summary

Mathematical Knowledge for Teaching is a fairly new construct. This review of literature revealed relatively few studies on the role of professional development in developing MKT. The research cited here did, however, give insight to the potential professional development settings hold in developing aspects of MKT. In the following section a description of professional development programs and designs will be shared.

Professional Development

In 2000, the National Council of Teachers of Mathematics (NCTM) published its *Principles and Standards for School Mathematics*, which provides a vision for school mathematics. Of these principles, the Teaching Principle states that effective teaching requires knowing and understanding mathematics. This principle also insists that teachers know and understand their students as learners; therefore, they must possess strong pedagogical strategies and provide students with a challenging and supportive classroom environment. Additionally, they must continually seek professional improvement (NCTM, 2000). For classroom teachers, professional development is their means to acquire this knowledge and understanding. Determining the impact of professional development on teachers is a difficult task, but one that must be explored (Smith, 2001). Smith stated the following:

> Considerable time, energy, and financial resources are being expended on professional development efforts that are not effective. To change this practice, we must continually document and evaluate our efforts so that we more fully understand the programs that make a difference in classroom practices and in the lives of students. (p. 57)

The following sections describe various components of professional development programs. The information is organized according to whether its components are ineffective or effective.

Ineffective Professional Development

For most teachers professional development is often conducted, and there is no evidence that the teachers and students are changed in any way by the experience. The teachers simply return to their classrooms after attending mandated district staff development or elected workshops and never make changes to their instruction. These types of professional development approaches merely treat teaching as routine and often do not relate specifically to the mathematics teachers must know and teach (Smith, 2001).

Furthermore, the professional development many teachers receive is not properly preparing them to teach for student achievement, mainly because it occurs in three limiting forms. The first form is the lack of sustained professional development that helps to broaden teachers' content knowledge base. This type of professional development does not include a follow-up to show evidence that the teachers or students are changed in any way by that experience. The second form is professional development mandated by the district. This type of professional development often treats teaching as being routine. Instructional changes and positive impacts to student achievement are rarely seen as a result. The third form of professional development does not specifically relate to the mathematics teachers must know and teach (Smith, 2001). For example, mathematics teachers may elect to attend a professional development session that is unrelated to what they teach just to receive credits towards licensure. These forms of professional development fail to achieve the ultimate goal of student achievement (Smith, 2001). Alternatively, professional development is needed that will transform how teachers acquire the knowledge necessary for the teaching and learning of mathematics. Therefore, new forms of professional development are needed that can enhance teachers' knowledge so that ultimately they will have a positive impact on student achievement.

Effective Professional Development

Professional development leaders have recognized the limits of some professional development models and acknowledged the role that professional development plays in increasing student achievement (Whitcomb, Borko & Liston, 2009). Hence, research has focused on different models. For these reasons, the next sections are devoted to two effective professional development models, namely the immersion experience and practice-based professional development.

The Immersion Experience. Loucks-Horsely, Love, Stiles, Mundy, and Hewson (1998) suggested the immersion experience as one way to change professional development practices. Grounded in adult learning research, the immersion experience is a direct experience with mathematics content and the process of problem solving. In this type of professional development setting, teachers learn mathematics content and processes of problem solving at their own level because they construct their own meanings of the ideas from their prior experiences. Additionally, immersion experiences provide opportunities for teachers to be the learner of the mathematics. The benefit for teachers as learners is that it broadens their own understanding and, often, dispels any misconceptions. This experience provides teachers with both content and skill knowledge that is vital for teaching mathematics. Also, immersion experiences prepare teachers to implement instructional changes in their classroom because they have experienced the mathematics for themselves and have been reflective about the learning during the process (Loucks-Horsley et al., 1998).

While Loucks-Horsley et al. (1998) discussed professional development experiences in which teachers' working mathematical tasks increased their knowledge for teaching, they also cautioned that this one aspect of professional development was not enough to increase teachers' knowledge for teaching mathematics. To this end, it is necessary to examine other types of professional development programs.

Practice-Based Professional Development. Smith (2001) describes practice-based professional development as a program situated in practice. Specifically, the program should include the examination of materials taken from classrooms. An example of this is a mathematical task accompanied by a set of student responses to that task. Within mathematics education research, practice-based professional development supports the transformation many teachers of mathematics need (Smith, 2001). By providing teachers with opportunities to develop new mathematical knowledge through practice-based strategies, this model allows teachers to develop new techniques for instruction.

In planning practice-based professional development for teachers, McDuffie (2003) studied the impact of the work on one classroom teacher who wanted the opportunity to experience transformative learning. Over the course of one year, the researcher and classroom teacher collaboratively planned and reflected on the lessons taught by the classroom teacher. Several trends emerged from this study. First, when a specific class or aspects of practice were the main focus of the training, changes to instruction were manageable. Second, as small changes occurred, other aspects of the instruction were affected, in this case the technique of questioning. Finally, when programs were situated in classroom practice, the teacher was able to make connections to her own practice and implement ideas from research with more flexibility (McDuffie, Mather, & Reynolds, 2003). In summary, this case study allowed one teacher to experience for the first time transformative changes to instructional practice. Similar to how students learn, this case study supports that teachers also learn best when they have a self-identified need.

Like McDuffie (2003), Carpenter and colleagues (1999) sought to engage teachers in practice-based professional development. The Cognitively Guided Instruction (CGI) research program focused on three integrated components. Of those three, the component most relevant to this review of literature was on the way teachers' knowledge, beliefs, and practices were influenced by their understanding of students' mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Therefore, Carpenter and colleagues designed CGI to help teachers understand how to navigate through students' mathematical thought processes. CGI is centered on the idea that when teachers implement practices such as listening to students and making sense of their reasoning, students are provided with a better education than if those practices are not implemented in the classroom. The research studies examined as a result of that process were two-fold. First, they yielded results that when teachers learned to examine students' work while in a professional development setting, the discussion was focused on the kinds of strategies students used. Additionally, the teachers hypothesized about the students' understanding and misunderstandings, the kind of instruction that students may have received, and the type of instructional changes needed to resolve misunderstandings. Second, this program strongly impacted those who design professional development programs. As a result of this work, mathematics educators recognized that professional development with a goal of influencing teachers' knowledge, beliefs, and practices could help teachers understand students' mathematical thinking and over time lead to instructional changes and student achievement (Carpenter et al., 1999).

Professional Development Summary

The Teaching Principle as set forth by NCTM clearly states that teachers must have a strong knowledge base about the teaching and learning of mathematics (NCTM, 2000). In the field of education, professional development is the means for teachers to obtain that knowledge. The work of Smith (2001), however, exposes the deficiencies in many of the professional development programs in which teachers are obligated to attend. In an attempt to combat those deficiencies, Loucks-Horsely et al. (1998), McDuffie (2003) and Carpenter et al. (1999) have presented research focusing on two forms of professional development that both proved to be effective. Considering these two types of programs can help professional development leaders implement more effective programs and ultimately support teachers as they acquire the knowledge they need to teach mathematics.

Summary

In the *Second Handbook of Research on Mathematics Teaching and Learning*, Sowder (2007) discussed the goals of professional development. Of the six goals she recommended, two are worthy of discussion in this literature review as they offer suggestions for developing mathematical content knowledge and pedagogical content knowledge. First, Sowder recognized that knowledge about mathematics is important not only to the education community, but also to society as a whole. Additionally, she recognized that technology has forever changed the landscape of learning mathematics. Therefore, the knowledge teachers need to have in order to teach mathematics is vital. Unfortunately, policymakers underestimate the knowledge needed for teaching mathematics. The need for teachers to know mathematics differently has been recognized within the educational community for years (Ball et al., 2008; Hill et al., 2004; Ma, 1999) but researchers are just now beginning to explore how teachers can develop that type of knowledge. The second goal, developing pedagogical content knowledge, aligns with the ideas previously discussed from Shulman (1986). This type of knowledge is multidimensional in that it should include the knowledge about how students learn, knowledge about teaching, and knowledge of the curriculum. Now that research on professional development has uncovered what teachers need to be effective, this knowledge should guide the development of programs for teachers (Sowder, 2007). Therefore, programs that develop that knowledge are important for teachers. To this end, the significance of this study lies in its examination of the potential of professional development to focus teachers' emerging Mathematical Knowledge for Teaching.

The next chapter describes the methodology used in this study to examine Mathematical Knowledge for Teaching in two professional development settings.

CHAPTER III: METHODOLOGY

Introduction

Programs that develop teachers' Mathematical Knowledge for Teaching are important because they ultimately lead to student success (Kazemi et al., 2009). Furthermore, the research community has recognized that teachers need to know mathematics differently in order to teach effectively (Ball et al., 2008; Hill et al., 2004; Ma, 1999). Therefore, the means in which teachers develop the type of knowledge needed to teach mathematics must be examined.

This chapter begins with a description of the design of the study. Following the description of the design is a restatement of the purpose of the study and the research questions. This section is followed by a description of the school settings and participants. Next, a description of the instruments used in the study is provided, followed by the procedures related to the study. Finally, methods for data analyses are reported for all qualitative data.

Design

This study utilized a qualitative approach to examine the domains of Mathematical Knowledge for Teaching that emerged in two different professional development settings. Qualitative means were employed due to the nature of building upon current phenomenon, namely the construct Mathematical Knowledge for Teaching. Specifically, the researcher utilized the modified analytic induction approach. In describing this approach, Wiersma (1995) stated the following:

In this approach, the researcher starts with specific research question(s); identifies

> virtually all instances (or cases) of the phenomenon under investigation; and investigates each case, employing an iterative process where the research question or phenomenon explanation is revised until he or she arrives at a suitable comprehensive, descriptively rich narrative. (p. 219)

Based upon the recommendations of Wiersma, the researcher identified two specific research questions. While conducting the study, the researcher identified instances of the phenomenon, MKT, under investigation until she could describe instances in which the phenomenon was observed.

Purpose of Study

The purpose of this study was to examine the domains of Mathematical Knowledge for Teaching that emerged in two professional development settings. The following research questions were developed to guide the focus for examining this construct.

Research Questions

1. What domains of Mathematical Knowledge for Teaching emerge in a professional development setting centered on teachers' completion of mathematical tasks?
2. What domains of Mathematical Knowledge for Teaching emerge in a professional development setting centered on teachers' analysis of student work?

Sample

The sample for this study was taken from two schools located in rural communities in north-central Mississippi. The schools were purposefully selected due to their need for increased student achievement in mathematics and convenient location to the university. In the following paragraphs, a justification for the selection will be provided.

School A

According to the Mississippi Assessment and Accountability Reporting System (Mississippi Department of Education, 2010), School A had approximately 398 students in grades three through six. School A's accountability status was "Academic Watch" due to not meeting No Child Left Behind (NCLB) Adequate Yearly Progress nor Title I improvements. This status was based on students' performance on the Mississippi Curriculum Test Part 2 (MCT2). Table 1 provides a synopsis of School A's mathematics scores on the MCT2 for the 2008-2009 school year.

Table 2

School A MCT2 Math Scores

Grade level	Number tested	% Minimal	% Basic	% Proficient	% Advanced
3	105	2.9	28.6	46.7	21.9
4	93	9.7	31.2	55.9	3.2
5	107	15.0	25.2	52.4	7.5
6	93	8.6	21.5	54.8	15.1

Table 2 illustrates how the students' performance on the MCT2 in mathematics was not improving. For example, the percent of students scoring advanced was decreasing from one grade to the next while the percent minimal was increasing. This table provides a picture of the need for this school to make improvements in the area of mathematics. To this end, professional development for teachers at this school was vital.

School B

According to the Mississippi Assessment and Accountability Reporting System (MDE, 2010), School B had 180 students in grades three through six. School B's accountability status was "At Risk of Failing" due to the failure to meet NCLB Adequate Yearly Progress or Title I improvements. The school's academic status was based on their students' performance on the MCT2. Table 3 provides a synopsis of School B's mathematics scores on the MCT2 during the 2008-2009 school year.

Table 3

School B MCT2 Math Scores

Grade level	Number tested	% Minimal	% Basic	% Proficient	% Advanced
3	42	16.7	64.3	16.7	2.4
4	38	10.5	36.8	50.0	2.6
5	59	42.4	25.4	28.8	3.4
6	41	17.1	43.9	36.6	2.4

Table 3 illustrates how the students' performance on the MCT2 in mathematics was not improving. For example, a majority of the students scored basic in the third grade and a majority of fifth graders scored minimal. The low performance at these grade levels demonstrated a need for this school to make improvements in the area of mathematics. To this end, professional development for teachers at this school was also vital.

Participants

Individuals invited to participate in this study were mathematics teachers in grades three through six from each of the previously described schools. In addition to regular classroom teachers, special education teachers were invited to participate. The inclusion of special education teachers was appropriate for three reasons, namely, they were the resource teachers, their work in the classroom directly impacted student achievement, and they, too, needed Mathematical Knowledge for Teaching. Additionally, their perceptions about the teaching and learning of mathematics offered a unique aspect to this study. In the following paragraphs a description of each school's participants will be provided.

Participants from School A

Teachers from School A who were invited to participate in the study were elementary mathematics and special education teachers working in grades three through six. School A had three teachers teaching mathematics in third grade, three teachers in fourth grade, three teachers in fifth grade, two teachers in sixth grade, and two special education teachers. All thirteen teachers were female. Of the school's thirteen teachers eligible to participant in this study, only six agreed. Five participants were Caucasian females with only one teacher being an African-American female. All six were certified in K-8 Elementary Education, with one participant who had received her national board certification and one with a master's degree. The participants ranged in number of years of teaching experience from one to thirty years.

Participants from School B

Teachers from School B who were invited to participate in this study were elementary mathematics and special education teachers working in grades three through six. School B had two third grade teachers, three fourth grade teachers, two fifth grade teachers, one sixth grade teacher, and two special education teachers. All ten teachers were female. Of the school's ten teachers eligible to participate, only five agreed. All five were African American females with a range of years of teaching experience from one to thirty years. Four of the five were K-8 Elementary Education certified with one participant 7-12 Mathematics certified. One participant from School B had received her national board certification.

School Settings

All sessions conducted at School A were in one participant's classroom. This setup allowed for the participants to be seated in desks, which were typically grouped in pairs. The professional development leader utilized the classroom's white board as a means for recording participants' ideas and was able to walk around observing participants during small group discussions. At each session and at all times, the researcher was seated away from, but within hearing distance, of the participants. The video recorder was positioned next to the researcher. Figure 1 displays the room layout for School A.

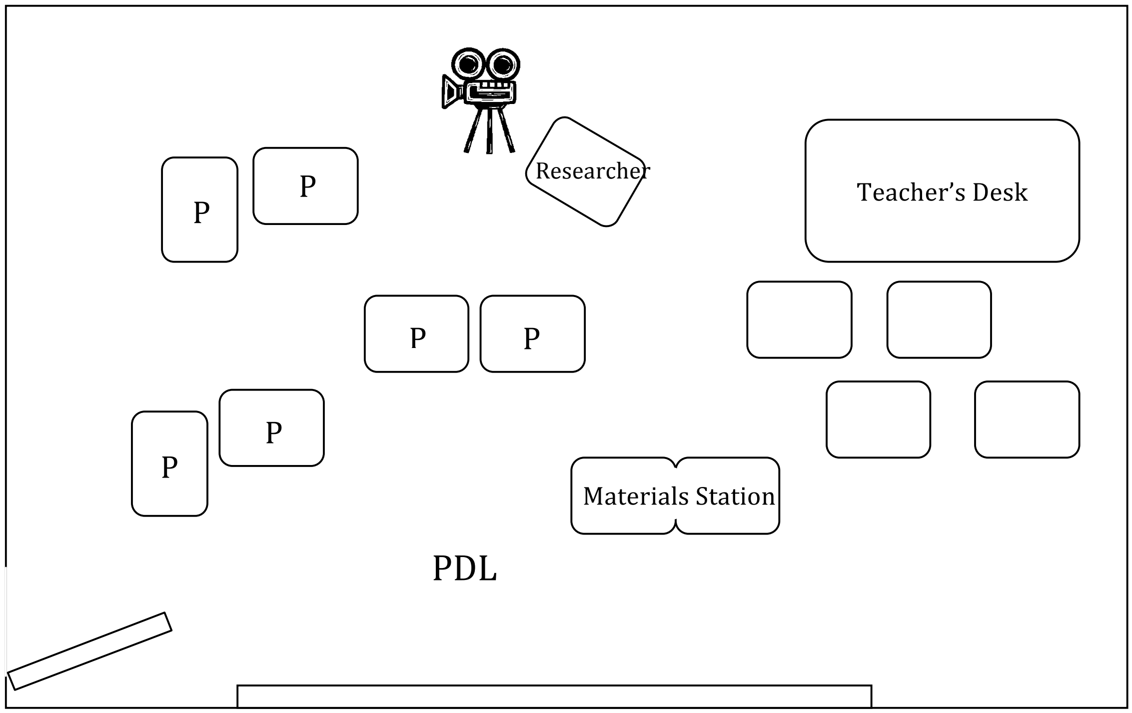


Figure 1. The floor plan for School A.

Professional development sessions conducted at School B were conducted in the school's teacher workroom. In this room, only a "kidney-shaped" table and five chairs were available. Therefore, participants sat on one side of the table while the professional development leader stood on the other side. A white board was also used in School B. At each session and at all times, the researcher was seated away from, but in hearing distance of, the participants. The video recorder was positioned next to the researcher. Figure 2 displays the layout of the room at School B.

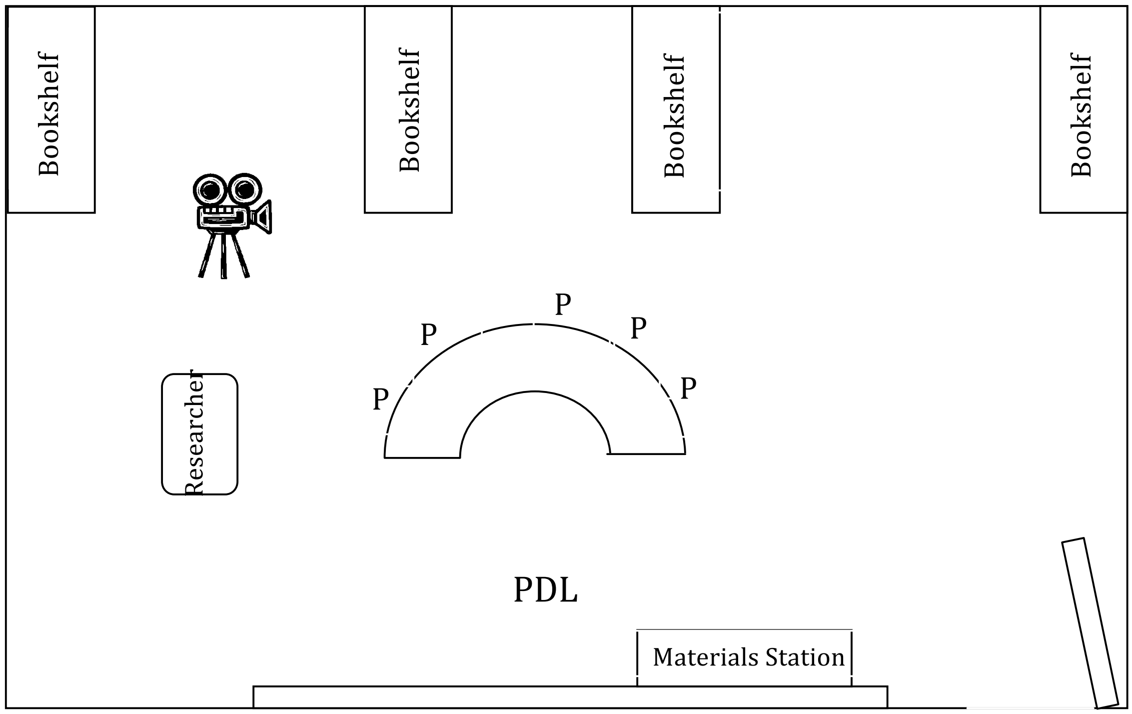


Figure 2. The floor plan for School B.

Instruments and Data Sources

The researcher utilized three instruments in this study to capture the domains of Mathematical Knowledge for Teaching in the two professional development settings. The three instruments were participant weekly reflections, observation guides, and the researcher. In addition to these instruments, professional development session transcriptions served as an additional source of data. In addition, the researcher served as a tool as she observed and recorded ideas from within each setting. Each instrument will be described in the following paragraphs.

Participant Weekly Reflections

At the conclusion of each professional development session, participants responded in writing to a prompt. The researcher and the professional development leader created the prompt collaboratively. The prompts allowed the participants' personal reflections about Mathematical Knowledge for Teaching to be captured. Additionally, the prompts captured the participants' perceptions of the professional development setting. Appendix A contains an example of a prompt used.

Observation Guide

The researcher utilized an observation guide during each professional development session as a means for recording field notes when participants were engaged in small group discussions. Due to the researcher's current employment position, she was able to pilot this instrument prior to data collection for this study within other professional development settings. The researcher attended a series of three professional developments. In each setting, she utilized the tool while observing a small group of teachers interacting. After each observation, the researcher discussed the process with her advisor, made changes to the guide, and utilized it again until the researcher was satisfied with its final form. This process allowed the researcher to design the most efficient tool to be utilized during small group conversations for this study. Appendix B contains an example of the observation guide in its final form.

The Researcher

The researcher served as an instrument for this qualitative study. The researcher was a Caucasian female pursuing a doctorate of education with an emphasis in elementary mathematics education. The researcher held a Master of Education in Curriculum and Instruction and had four years of classroom teaching experience in grades four through seven. Additionally, the researcher had taught methods in mathematics courses for the university in which she was earning her doctorate. The researcher also worked extensively on two externally funded professional development projects at the university. Her specific roles on those two projects included, but were not limited to, engaging in-service teachers and their classroom students in standards-based instruction. These experiences provided the researcher with the knowledge base necessary to serve as an instrument in this study.

Professional Development Session Transcriptions

The observation guides offered instances of Mathematical Knowledge for Teaching within small groups; however, the researcher also sought to capture whole group instances. Therefore, each professional development session was video recorded. The camera was stationed to the side and behind participants in School A and to the side and away from participants at School B. At the completion of the video collection, the researcher transcribed each session. The sessions were transcribed verbatim with pauses, etc., noted. The transcriptions captured instances of Mathematical Knowledge for Teaching within the whole group from each setting.

Procedures

The researcher sought approval from the dissertation committee. Upon this approval, the researcher sought the approval of the Institutional Review Board (IRB). Once received, the researcher visited each school site for the purpose of inviting teachers to participate in the study. This initial visit served as an informal meeting with the third through sixth grade mathematics and special education teachers. During this meeting, an expert in the field of mathematics education led teachers through a mathematical task purposefully selected to encourage teachers to participate and to demonstrate the type of work in which they would be engaged during the professional development. The expert served as the professional development leader throughout the study. Her qualifications can be found in Appendix C.

At the completion of the task, the researcher described the study, shared the IRB information sheet (see Appendix D), and provided the teachers with a schedule of the professional development sessions. Professional development sessions were held after school at each site for 1.5 hours. Prior to the first meeting, the researcher planned each session with the assistance of the professional development leader. Appendices E and F contain samples of the professional development plan for the initial immersion and practice-based sessions, respectively. As the professional development sessions transpired, the two met approximately four hours weekly to revise and discuss each session.

Prior to beginning data collection, participants engaged in two professional development sessions. These sessions helped the researcher and professional development leader establish the sociomathematical norms (Rasmussen, Yackel, & King, 2003). Beginning during the week of October 25, 2010 and continuing for four weeks afterwards, data collection for this study was conducted. In order to capture MKT during each session, the researcher utilized the session transcriptions, participant weekly reflections, and an observation guide, as previously described.

At School A, the professional development utilized the immersion experience for the first two sessions in which data was collected. During the third and fourth sessions, participants at School A were engaged in the practice-based professional development. At School B, the practice-based professional development was utilized for the first two sessions in which data was collected. During the third and fourth sessions, participants at School B were engaged in the immersion experience. At both sites, the first two sessions focused on the multiplication of fractions, while the third and fourth sessions focused on division of fractions. Appendix G contains an outline of the professional development experience in which the school was engaged and the topics which were discussed during the four weeks of data collection.

Implementing both the immersion experience and practice-based professional development at each school allowed for participants to be engaged in discourse surrounding mathematical tasks and in discourse surrounding student work associated with the mathematical tasks. Thus, the professional development all participants received was rooted in both effective forms of professional practice (Smith, 2001). Furthermore, the researcher elected to alternate the type of experience in which the participants were engaged during the actual data collection. This decision was made because alternating the type of professional development in which the participants were engaged helped to eliminate attendance issues and rejuvenated their interest in the work.

As previously stated, sessions for each school included the multiplication and division of fractions. The following section provides a brief justification for the use of fractions as the focus of the professional development sessions.

The Case for Fractions

The purpose for utilizing fractional concepts in this study was two-fold. First, fractions are a crucial concept in elementary school mathematics. Smith (2002) stated that “no other concept is as mathematically rich, cognitively complicated, and difficult to teach as fractions.” (p. 3). When teachers of elementary school mathematics invest their time building conceptual meaning for fractions, student understandings increase (Cramer & Henry, 2002). Instructional approaches to the teaching of fractions, however, have been heavily symbolic and procedural. These routine practices have left students without a solid foundation for fractional concepts (Kamii & Warrington, 1999). In response, Huniker (2002) stated, “It is time to shift the emphasis and redefine the goal of fraction instruction in elementary school from learning computational rules to developing fraction operation sense” (p. 78). The best means for providing teachers with new instructional practices is through professional development rooted in the very essence of these ideas (Huinker, 2002). Given the complexity of the concept and that there is a need for instructional change in the way fractions are taught, the focus of the professional development for this study was on fractions, specifically the multiplication and division of fractions.

Second, immersing participants in fractional work and discussing classroom artifacts of that fractional work helped reveal mathematical knowledge above and beyond that which is common. Therefore, working with fractions lent itself to highlighting more than one domain of the MKT model.

Data Analysis

The researcher utilized the six domains of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008) to frame the coding scheme of the data generated from each of the three instruments. The modified analytic induction approach of analyzing the data according to the MKT framework enabled the researcher to examine trends within the two settings related to the phenomenon under investigation (Patton, 2002; Wiersma, 1995). This iterative process will be discussed in the following sections.

Phase 1

The researcher began transcribing video recorded sessions, approximately two weeks after the final professional development session. The researcher spent two weeks engaged in the transcription process. The transcription of each video took approximately four hours. Upon completion of the transcription process, the researcher removed herself from the data for two weeks, allowing her to approach the analysis of the data without bias.

To begin analyzing the data, the researcher started by coding the video recorded sessions. As the researcher coded, she identified instances of MKT domains. These instances were highlighted and then labeled accordingly. The coding of the session transcriptions took two days. Next, she utilized the same process for the participant weekly reflections. This coding process took one day. Finally, the researcher coded the observation guides. The researcher spent one day coding the observation guides. At the completion of the entire coding process, the researcher again removed herself from the data for approximately three weeks, allowing for other commitments to be met. When the researcher returned to the data, she discussed her preliminary results with her advisor. The initial coding process was limited in scope, as the researcher focused on mathematical and pedagogical misunderstandings of the participants. Given the limited results and time away from the data, her advisor suggested the researcher re-examine the data. The advisor also reminded the researcher that the focus of the data was to reveal the types of MKT domains around which the participants were focusing their ideas while engaged in either the immersion experience or practice-based professional development. With this advice in mind, the researcher began Phase 2 of the data analysis process.

Phase 2

The researcher began the recoding process approximately twelve weeks after the final professional development session and six weeks after the completion of the first phase of data analysis. As the researcher recoded the session transcriptions, she focused on the type of MKT that was revealed through participants' discussions rather than gauging the accuracy of the mathematics. With this lens, the researcher found more instances of MKT being revealed. The researcher again discussed the preliminary results with her advisor. The advisor guided the researcher in reporting these new findings.

Limitations and Delimitations

Due to this research study's qualitative nature, the results are not generalizable. Identifying the sample that allowed the researcher to explore Mathematical Knowledge for Teaching in two separate professional development settings resulted in purposeful sampling. Due to the use of purposeful sampling, however, the results of this study are not generalizable. Additionally, the researcher served as the primary instrument for data collection, primary developer of the session plans utilized within the professional development settings, and primary data analyst. Therefore, a third limitation involved the potential for researcher bias. In an attempt to decrease bias, the researcher separated herself from the data after the completion of transcribing and conducted two phases of analysis.

Summary

Students' mathematical successes are directly linked to the Mathematical Knowledge for Teaching that teachers possess (Kazemi et al., 2009). How Mathematical Knowledge for

Teaching is developed within teachers, however, is not clear. Hence, this study examined how two different professional development settings elicited discussions related to the domains of Mathematical Knowledge for Teaching. The participants were third through sixth grade mathematics teachers from two different schools within the same northcentral Mississippi county. Data collection consisted of participants completing weekly reflections of each professional development session, researcher field notes to capture small group discourse of Mathematical Knowledge for Teaching, and transcriptions of whole group discourse of Mathematical Knowledge for Teaching. After data collection, the researcher engaged in two phases of data analysis. Each time, the six domains of Mathematical Knowledge for Teaching were utilized to frame the coding scheme. The results are provided in Chapter IV.

CHAPTER IV: RESULTS

Introduction

To obtain both mathematical content knowledge and its supporting pedagogical knowledge, teachers must look to professional development programs. Professional development programs for teachers hold the potential for transforming the way in which mathematics is taught (Smith, 2001). Such programs should provide teachers with inventive ideas about the teaching and learning of mathematics (Smith, 2001). Per the recommendations of Smith (2001), two forms of professional development have been found to be effective, namely the immersion experience and practice-based experience. In an effort to contribute to this research, this qualitative study examined the type of knowledge, specifically Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008), that emerged in two professional development settings. The significance of this study, therefore, was in its examination of how different foci of professional development revealed different domains of Mathematical Knowledge for Teaching.

In this chapter, the results of the data analysis will be presented in two major sections. The first section will contain the results from the immersion experience and will include two subsections. In the first subsection, the results from the session transcriptions, participant weekly reflections, and observation guides taken from the professional development focusing on the immersion experience along with the MKT domains that emerged will be shared. The second subsection will be a response to Research Question 1. The second major section will be the results from the practice-based experience and will include two subsections. In the first subsection, the results from the session transcriptions, participant weekly reflections, and observation guides taken from professional development focusing on the practice-based experience along with the MKT domains that emerged will be shared. The second subsection will be a response to Research Question 2. Throughout the chapter, data taken from session transcriptions and participant weekly reflections are used to represent common Mathematical Knowledge for Teaching ideas that occurred across participants. The data taken from the tools differ, however, in terms of the clarity with which the participants' ideas are articulated. Finally, a chapter summary will be shared.

The Immersion Experience

Loucks-Horsely, Love, Stiles, Mundy, and Hewson (1998) described the immersion experience as a direct experience with mathematics content and the process of problem solving. In this type of professional development setting, teachers learn mathematics content and processes of problem solving at their own level. Additionally, immersion experiences provide opportunities for teachers to be the learner of the mathematics. The next three sections contain the results yielded from the session transcriptions, participant weekly reflections, and observation guides from participants engaged in the immersion experience.

Session Transcriptions

School A–Immersion 1. School A's first immersion professional development session focused on exploring the models for the multiplication of fractions. One vignette will be shared to capture how various MKT domains were revealed during this session.

In the course of School A's first immersion task, i.e. Mary's Casserole, participants were asked to represent their work with a picture and number sentence. Upon the completion of the task, the groups were asked to display their work at the front of the room and then participants were to decide which representation best depicted the area model for multiplication. The following dialogue ensued when the professional development leader asked if there was a difference between two of the representations that the group had decided were the best depictions of the area model for multiplication. All real names have been replaced with pseudonyms and the professional development leader is identified as PDL.

PDL: Okay, so I put the question out there about what is the difference between the first two, and are they both an area model. You know what's going on? Is there a big difference between the two? Is it a big deal? Erin, your card came up. So what were y’all saying?

Erin: We said that they were both area, but the second one the students would react better cause it’s easier to see the area, you see more length and you see more width than you would on the first one.

PDL- You're thinking that because this one, you know length and width you can see it, but over here it’s so narrow that they may not see it?

Erin: Uh-huh, they wouldn't realize that that would be one by six, as well as they would a two by three.

PDL: Okay. All right. Lindsay, what were you'll saying?

Lindsay: Well, I mean, I just thought that the second was so much truer to what the area model is that to me, but I'm third grade and I don't have near as much experience as some of these but I just didn't know that that first one would be the area model.

PDL: Okay, so we're all kinda leaning this way but we're not really sure why not this one (pause). We're thinking that maybe kids like it but mathematically there ought to be a reason why we should be here and not here. Here's (pause)

Lizzy: It looks more like an array (pause)

PDL: Why do you say an array?

Lizzy: Because it is one across but they're more about counting one across or two across. That's six across and it is one down but it would be hard for mine to visualize that as an area model.

In this vignette, the participants were attempting to decide which representation accurately represented Mary's Casserole using the area model for multiplication. As the professional development leader prompted them to justify their selection, participants began focusing on students as opposed to providing a mathematical justification. This way of thinking began with Erin, who in lines 5 and 6 justified her choice based on her perception of students' reactions. Building on this, Lindsay and Lizzy also appeared to be thinking about students, specifically their own students. In lines 13 through 16, Lindsay stated that she was working with students in third grade. Similarly, Lizzy in lines 22 and 23 stated that her students would experience difficulty with the area model. As the participants justified their mathematical decisions based on their understandings of students, they provided evidence of KCS being the focus of the professional conversation.

School A–Immersion 2. The second immersion professional development session in School A focused on the algorithm for the multiplication of fractions. Three vignettes will be shared, because collectively they captured how various MKT domains were revealed during this session.

To start the second immersion session, the professional development leader asked participants to examine three contextual multiplication problems and decide which one best depicted the area model for multiplication. From this question, the participants prompted a discussion regarding key words. The following dialogue is that discussion as it ensued.

Amber: When I was in school they told me that “of” was a key word for multiplication and the [word problems] that you can't actually see actually use the word “of” so that might be a way for the kids to see it as multiplication if they don't know area. Does that make since?

PDL: So, when it involves area you think of multiplication?

Amber: Uh-huh.

PDL: But then you've noticed in the other, is it both of the others?

Amber: Uh-huh. In both the others it says, “one third of” or “one fourth of”

PDL: “Of” and that's linked to multiplication (pause). So, Lindsay, what did you say to her when she told you about that?

Lindsay: I was like, “Oh!”

PDL: You liked that?

Lizzy: Well, it works.

Lindsay: I liked it. I think “of,” yeah. I mean because well, it really does make sense.

Lizzy: I teach “of” means to multiply. “Of something.” Whether it’s “of a fraction” or “of a whole number.”

PDL: Rachel, what do you think?

Rachel: Same.

PDL: Gosh, y’all are easily convinced. Erin, what are your thoughts on that?

Erin: I mean the ones we've looked at it works on.

PDL: Oh, wait a minute, why are you couching that with the ones we've looked at?

Erin: Because I'm sure you could find ones that it doesn't work on.

PDL: Okay, so yeah, we do need to be careful about that. We've talked about this before, right, Amber? So in terms of student learning, and I want y'all to talk about this in your pair, in terms of student learning, what are the ramifications of using this model and what are the ramifications about using this as a means for understanding multiplication? We have some problems that promote this way and some that promote this way so take just a minute to talk about the ramifications, pros and cons, consequences of this way and this way. Okay, so just a minute to talk about that. (Participants talked to their partner for 2 minutes.)

PDL: Okay, so some ideas. We have seen where some word problems elicit this idea of area which in turn gets you multiplication. In some word problems we have that word "of" that gets you thinking about multiplication. So my question was what would be the pros and cons for each of these models, and so we'll start with Lindsay. Lindsay, something you and/or your partner were saying related to this?

Lindsay: Well, with the word problem they have to actually read the problem and use the word "of" in the context. They have to, um, you know, using "of" then you can't visualize the area model then the "of" would work.

Amber: I'm trying to expand on key words because kids wanna think a key word means, we have to multiply and that's not how key words work. You have to use it in context of what the whole problem says. It's just a signal that means you need to watch out you may have to multiply here. They have to read it through and see what it means. They can't just go through and say okay, it says "each" so we multiply. They have to use it in context but kids don't want to do that. They want to say there is the word “of” so I multiply. So that is one of the bad things of key words. They think its just look at the key word and do what it says.

Throughout this portion of the session, the participants were focusing their ideas about contextual multiplication problems around the use of teaching key words. As the discussion unfolded, the participants justified why they agreed this technique was a good approach to teaching multiplication word problems. In line 1, Amber began discussing her own knowledge of using the word “of” to solve multiplication problems. Additionally, she reflected on how she taught key words in her classroom. Her thoughts about key words reflected CCK, because many people, regardless of their profession, believe key words are a valid problem-solving strategy. In line 16, Lizzy stated that she also taught this technique in her classroom. Like Amber, this revealed Lizzy’s CCK but also her KCT as she, too, focused her thoughts on teaching. When the professional development leader pushed the participants to think about the pros and cons of teaching keywords to students, lines 24 through 47, a shift towards a discussion more focused on the potential misunderstandings about the strategy developed. This shift allowed the participants to consider the importance of students understanding the contextual problems, not just limiting them to keywords. Therefore, this vignette demonstrated how a progression from CCK to SCK evolved when participants were engaged in the immersion experience.

As this discussion unfolded even further, Erin and Alicia provided classroom examples from their teaching that supported the other participants’ previously made statements. The following segment from this session provides a glimpse of the remainder of the discussion.

Erin: Well, what we were saying sometimes, like what Amber said, they do become dependent on those key words when they see them. And Alicia said she gave a test and they went through and she had mixed problems and they divided everything cause they get use to the key words so they think it's gotta be this way and they don't pay attention to what that problem actually says to do. So it can be good or it can be bad if they don't use it in context.

PDL- So, Alicia, you've dealt with this in your class, in 5th grade?

Alicia: Yes.

PDL: So how do you think the area model might help you in your classroom?

Alicia: Ummmm (pause). I was just saying that they have to know, I mean they have to understand why they are multiplying or whatever operation they are using not to look at key words even though you want to point them towards that but like with division I hadn't really said anything to the 5th graders about key words I was just kinda. (pause) They went through and we were on division so every problem they got to they think it will be division so that's where I just gave them a test that was adding and subtracting and multiplying and I wanted to kinda see what they actually go through and understand and they are just not looking, we're not just dividing because we are on division, you really have to understand but I think that the area model would be good.

In this exchange, Erin and Alicia shared an example from Alicia’s classroom to illustrate how teaching keywords limited her students on an assessment. In doing so, two primary MKT domains, KCT and KCS, were revealed. These domains were highlighted in lines 57 through 66 in which Alicia referenced her classroom assessment practices.

After these ideas were shared, the professional development leader shifted the participants’ focus toward the algorithm for the multiplication of fractions. In doing so, she asked the participants to respond to the following sentence starter, "When multiplying fractions you can multiply straight across because . . ." After approximately three minutes of working in small groups, the professional development leader asked participants to share their ideas with the whole group. The following vignette contains the responses from Erin and Rachel as they shared their ideas.

Erin: I put, "because the denominator is the total number of parts and the numerator is the total number of parts shaded or overlap to each other."

PDL: Okay, and so let me say this for Rachel and Alicia, last week we noted that sometimes people shade one way and shade the other way so when she says overlapped she is talking about those up here. So just to clarify it, Erin, will you say it one more time?

Erin: Ummmm (pause). The denominator is the total number of the parts, the parts the whole thing is divided into. And the numerator is the total number of parts that share or overlap with each other.

PDL: How does that relate back to what Lizzy and Rachel were saying? They sounded really different.

Rachel: Because when she said the denominator was the whole area that was three times four, the whole area and then the part that was overlapped two times three that was the numerator of the part that was shaded of the whole.

In this vignette, Erin and Rachel justified how the algorithm for multiplication works. Erin's description was about multiplication problems in general, while Rachel's response was more specific to the actual problem they were solving in this session. Both responses to the sentence starter yielded a mathematical conversation. In lines 73 through 75, Erin restated her ideas, those reflective of an understanding of the algorithm that is beyond common knowledge. Additionally, in lines 78 through 80, Rachel utilized Erin’s thinking and applied it to the actual problem being worked, a conversation dominated by participants’ SCK.

School B–Immersion 1. School B’s first immersion professional development session focused on the models for the division of fractions. One vignette will be shared because it best captured the MKT domains that were revealed during this session.

The professional development leader began this session by having the participants recall the two models of the division of fractions, a previously taught concept. After participants shared what they knew and the professional development leader recorded that information on chart paper, participants were asked to represent and solve the Peach Tarts task. The professional development leader gave directions about how participants would discuss their work and then allowed time for them to work. After approximately ten minutes, the professional development leader had each participant display her work at the front of the room. Additional time was given for the small groups to decide which representation they preferred and what model of division it represented. The groups were discussing their multiple representations of the Peach Tarts task when the following discussion ensued.

PDL: Jill, what was it about it that made you stop and think, “No, it's repeated subtraction”?

Jill: Because I thought about how in my head I was actually pulling apart the pieces. And putting one, two here okay here's one tart. One, two here's another one. And so I realized that we really didn't, we really weren't given a set number of groups. We didn't know that to begin with. And we were pulling apart to figure out the number of groups.

PDL: Okay, so can I add that to our poster? Cause you said that with repeated subtraction we are trying to figure out how many groups we have. So I'm going to add that and I'm going to use orange (adds it to the chart paper). Okay, so that's a nice observation. We're looking for the number of groups. Did y’all all do the same thing or were there any differences between the posters? Jill, did y’all notice anything different about them?

Jill: Uh-huh. Peggy went through and she shaded two, two, two, and then went back at the end and counted how many were left over and put those together. But I did my two together. And I did everything together and I didn't go back to the end to see what was left because I did everything first. And I didn't have anything left over.

PDL: Can you explain to me, y’all may understand this, these ones and twos? And if you need to come point that will be great.

Jill: Okay. All right, one part, two part, gone (shows this with her hands on her chart paper at the front of the room). One part, two part, gone. One, two, that’s one. One, two, that part is gone, there's one. Here's two, that one's gone.

In this vignette, one participant shared her ideas regarding repeated subtraction. Additionally, in line 3, she concentrated on the physical act of “pulling apart pieces.” Upon doing so, she provided the professional development leader with a better description of repeated subtraction. To this end, the professional development leader wrote that description on the chart paper used at the start of this session to capture the participants’ ideas about models of division. This exchange demonstrated how Jill’s reasoning elicited a mathematical response. As she described her work and the other participants’ work, lines 14 through 18 and lines 21 through 23, a mathematical understanding regarding repeated subtraction was revealed. Teachers demonstrating this type of knowledge are exhibiting SCK.

School B–Immersion 2. School B's second immersion professional development session focused on the algorithm for the division of fractions. One vignette will be shared because it best depicted the MKT domains that were revealed during this session.

The professional development leader began this session by having the participants provide their ideas about repeated subtraction, one model for the division of fractions. After she recorded the participants' responses on chart paper, the professional development leader asked the participants to revisit the Measuring Scoops task from the previous week. This time, however, they were told they had to represent and solve the task with the given manipulatives. After a lengthy and tedious conversation among the participants about that process, the professional development leader posed another question. Specifically, she stated, "So the problem we are going to do is six tenths divided by two tenths." The group was asked to represent and solve this problem using the given manipulatives. The following vignette is the whole group discussion which occurred after participants worked in small groups.

PDL: Okay, so coming over here now. I sorta see the same picture. Do y'all see their model? It's kinda the same, it's just broken apart. See the six tenths and the two tenths? But I'm wondering if, Stacy, you can tell us how y'all got an answer from that?

Stacy: Well, you weren't supposed to see [this representation with the given manipulatives], was she?

Jill: Uh-huh.

Stacy: Oh, you were supposed to see that (laughter).

PDL: (laughter) Well that helps us to see that was two tenths. And it kinda matched what they were thinking.

Stacy: Right. And so we did ten little cubes and then we broke six tenths off and this represented two tenths and so then we said one two-tenth, two two-tenths (pause) is that what we did? (giggling)

Jill: Three.

Stacy: And yeah that's it. Jill could have said it better.

Jill: No, she did just fine.

PDL: Jill, why don't you say it again so you can repeat what Stacy just said.

Jill: We started off with ten and first we took off six of the tenths to represent six tenths and then we took off two of the tenths to represent two, no, two of the tenths to represent two tenths and then we said we wanted to split these six tenths into two tenths so we had one, two, and three times that it could be split by the two tenths.

PDL: So when you think about what we were doing up here, what's the amount that you are taking out of the six tenths each time?

Carrie: Two.

PDL: The two tenths, so we're asking the question of, "How many two tenths are in six tenths?" And you all said that there were three. So there are three two tenths. So our answer is three. Why don't y'all take a minute to work that by inverting and multiplying to verify for yourselves that the answer is 3 (lots of laughter). Instead of working it in your heads. Cause I know some of you already did. What did y'all get?

Carrie: Three.

Rolanda: Three.

PDL: So you started out with six tenths of a whole unit and you found out that it took three two-tenths to fill that unit so that's "you're paying attention to the units" note. You know it's interesting. I've had teachers say to me, "Look I get how to do this by the algorithm but why if I start with six tenths and I divide it up, do I end up with more?" Cause they see that three as being more than the six tenths that they have here. But it's not. Why is it not? Why is that three not more than the six tenths?

Carrie: Cause it's three two-tenths.

PDL: Because it's three two-tenths. So to make sense of this stuff you have to make sense of the units and what's going on in there. Very nice.

In this vignette, Stacy and Jill grappled with describing their process for modeling and solving the question, "Six tenths divided by two tenths." In lines 18 through 22, Jill was able to describe the process of modeling the division of fractions in a specialized way. Hence, this portion of the discussion elicited SCK.

Participant Weekly Reflections

At the completion of each session, participants were asked a reflective question pertaining to the day's work (see Appendix A). The following two sections will capture these responses, first from School A and then from School B.

School A–Immersion 1. During this session, participants examined different representations for multiplying fractions. The overall goal was to introduce participants to the area model for multiplying fractions, recognizing this would support making sense of the algorithm. At the conclusion of the session, participants completed a participant weekly reflection that was designed to engage them in reflecting on the usefulness of the model as well as how they could utilize the information in the future. In the following paragraphs, two sample participant weekly reflections will be shared along with the MKT domains that were present in each reflection.

How is the area model helpful for thinking about multiplication of fractions?

Area models are very visual. I am a visual learner and so are many of my students. This will be most helpful not only to them but to all students because it makes it clearer.

Think about the information we have discussed today. How will you use this information in the future?

Well in 3rd grade I will not only have them model and interpret arrays but also area models. I will give them more area models when working with multiplication so that it will be a natural transition between representing multiplication facts to fractional parts when we move into fractions.

Figure 3. One participant's weekly reflection.

An examination of the first response in Figure 3 revealed that the participant was focusing on how the area model can be helpful for students working with the multiplication of fractions. In doing so, her ideas were focusing on KCS. In addition, in reflecting on how the information gained during the session would be used in the future, she described how she would utilize the area model in her third grade classroom. In doing so, her ideas were focusing on KCT. Taken from the same session, Figure 4 demonstrates how another participant's reflection elicited similar responses.

How is the area model helpful for thinking about multiplication of fractions?

I like the idea of shading not being necessary. I never could tell my students why the overlapping portion represents the product.

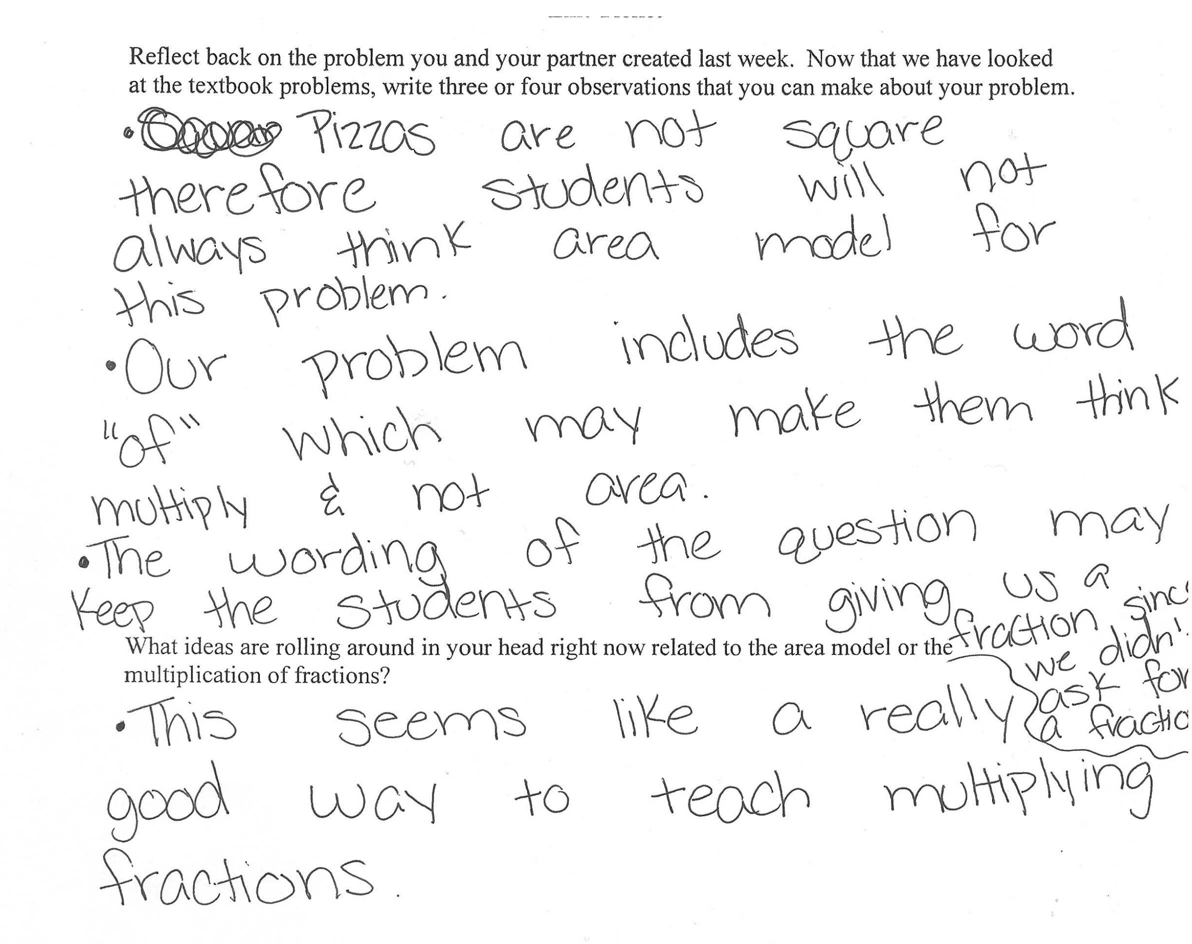
Think about the information we have discussed today. How will you use this information in the future?

When doing our models of multiplication, the models will be much more helpful. I can show students why it works, although this way, they probably won't need to ask. It makes sense.

Figure 4. One participant's weekly reflection.

Upon examining this reflection, the researcher noted that the participant's reasoning on the first part of the reflection exposed her ideas about the mathematics at hand. Specifically, this participant was reflecting on how she had never been able to provide her students with a mathematical justification for the area model. Therefore, the ideas she was presenting on this section of the reflection were focusing on students and content knowledge, a KCS response. In the second part, the participant was focusing her ideas on the teaching of this content. Specifically, she focused on using models to "show students why it works." Therefore, her response focused on the KCT domain.

School A–Immersion 2. During this session, participants continued their examination of the models for multiplying fractions while moving towards an understanding of the algorithm. At the conclusion of the session, participants completed a participant weekly reflection. This first question on the reflection was designed to engage the participants in reflecting on observations made about the word problems they wrote during the previous professional development session. In addition, the second question on the weekly reflection was designed to provide participants with the opportunity to reflect on ideas about the area model and multiplication of fractions. In the following paragraphs, two sample participant weekly reflections will be shared along with the MKT domains that were present in each reflection.


Reflect back on the problem you and your partner created last week. Now that we have looked at the textbook problems, write three or four observations that you can make about your problem.

- Pizzas are not square therefore students will not always think area model for this problem.
- Our problem includes the word "of" which may make them think multiply & not area.
- The wording of the question may keep the students from giving us a fraction since we didn't ask for a fraction

What ideas are rolling around in your head right now related to the area model or the multiplication of fractions?

- This seems like a really good way to teach multiplying fractions.

Figure 5. One participant's weekly reflection.

An examination of the weekly reflection contained in Figure 5 revealed that the participant was focusing on how the context of word problems can either support or fail to support the students' thinking about the targeted mathematics. She has described three aspects of the word problem previously created that most likely would not facilitate students' thinking about the area model. In doing so, her ideas were focusing on KCS. In addition, an examination of the second response revealed how the participant reflected on how the information gained during the session would be "a really good way to teach multiplying fractions." In this section of the reflection, her ideas were focusing on KCT. Figure 6, taken from the same session, demonstrates how another participant's reflection elicited different responses, and thus revealed different MKT domains.

Reflect back on the problem you and your partner created last week. Now that we have looked at the textbook problems, write three or four observations that you can make about your problem.

I didnt create problems last week, but ① you need to think of a square pizza to use area model. ② Your numerator would be the two pieces that overlap. ③ The word "of" is used in the problem. ④ Word choice for problem influences how students think about & answer problem

What ideas are rolling around in your head right now related to the area model or the multiplication of fractions?

Area model is introduced in third grade showing example  $\frac{2}{3}$ but it's rareley used to show how to multiply fractions. It should be tied into the area model for multiplying fractions soon after 3rd & used often. My students in 3rd loved area models.

Figure 6. One participant's weekly reflection.

In this reflection, the participant focused her ideas on the grade level in which students should learn the area model, the KCC domain. Additionally, this participant hinted at how this model should be used in the future. Specifically, she indicated that the area model should be used across grade levels, the HCK domain.

School B–Immersion 1. During this session, participants examined different representations for the division of fractions. The overall goal was to have participants model division of fractions, specifically using the repeated subtraction model for division. At the conclusion of the session, participants completed a participant weekly reflection. The first question allowed the participants to look at what common aspects of division word problems developed conceptual understanding of the division process for fractions. In doing so, this prompt provided the opportunity for the participants to think about KCS. In addition, the second question gave the participants the opportunity to describe the ideas they were focusing on after the session. In doing so, a true picture of the MKT domains on which the participants were choosing to focus would be revealed. In the following paragraphs, one sample participant weekly reflection will be shared along with the MKT domains that were present in the reflection.

How do the Peach Tarts and Measuring Scoops problems support students' thinking about division with fractions?

The problem allows students to think about peaches in parts because only 2/3 are needed to make a pie. Measuring scoops allows the kids to focus on the importance of units.

What ideas are rolling around in your head right now related to the division of fractions?

Units are important!!

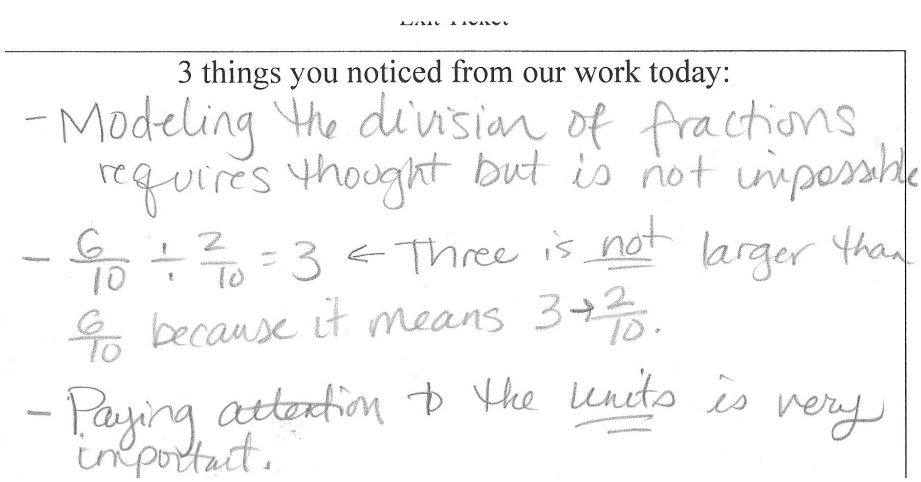
Repeated subtraction = finding the number of groups.

Partitioning = placing items in groups.

Figure 7. One participant's weekly reflection.

In examining this participant's reflection in Figure 7 the researcher noted the responses were centered around the mathematics. In the first question, the participant focused her response on student thinking about the content while emphasizing "parts" and "units." In the second question, the participant focused her response on the two models of division. Therefore, this participant weekly reflection led this participant to write about KCS and SCK.

School B–Immersion 2. During this session, participants continued their discussion about the different representations for the division of fractions while moving towards a conversation about the division algorithm. The overall goal, therefore, was to have participants develop the algorithm for division of fractions. At the conclusion of the session, participants completed a participant weekly reflection. This specific reflection required participants to provide three things they noticed, two questions they had, and one big idea from the session. Through this reflection, participants were provided the opportunity to express openly their thoughts or concerns on which they were focusing as a result of the professional development. In the following paragraphs, three sample participant weekly reflections will be shared along with the MKT domains that were present in each reflection.



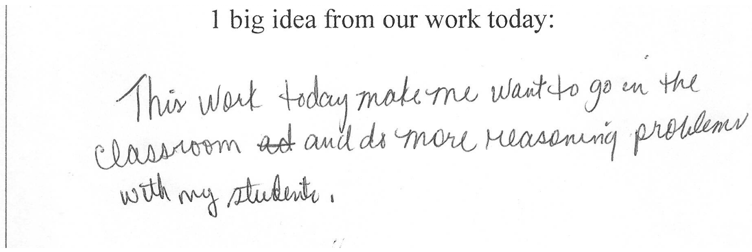
Exit Ticket

3 things you noticed from our work today:

- Modeling the division of fractions requires thought but is not impossible
- $\frac{6}{10} \div \frac{2}{10} = 3$ ← Three is not larger than $\frac{6}{10}$ because it means $3 \rightarrow \frac{2}{10}$.
- Paying attention to the units is very important.

Figure 8. One participant's weekly reflection.

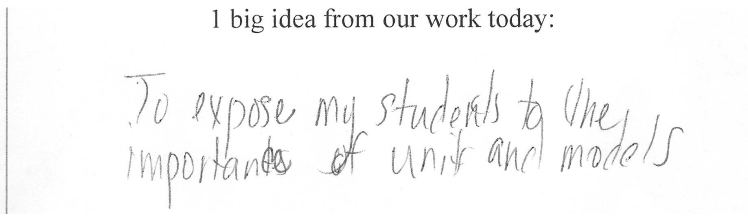
An examination of the reflection in Figure 8 revealed that the participant focused on three aspects of the work with the division of fractions, namely modeling, making sense of the quotient six tenths, and the importance of "units." The researcher noted that each of these were ideas that appeared in the session transcript. In this response, her ideas focused on a deeper understanding of the mathematical content, hence SCK. Figures 9a and 9b represent samples taken from the same session. They are two different participants' responses to the third part of this reflection.



1 big idea from our work today:

This work today make me want to go in the classroom ~~ad~~ and do more reasoning problems with my students.

Figure 9a. One participant's weekly reflection.



1 big idea from our work today:

To expose my students to the importance of unit and models

Figure 9b. One participant's weekly reflection.

An examination of Figure 9a revealed that the participant focused on how her work during the professional development session will impact her instruction with students. It was also worth noting that she was thinking beyond the mathematics and more about teaching mathematics through reasoning. An examination of 9b revealed that the participant had taken the important mathematical ideas from the session and thought about how it was important for her students to have those same ideas. In doing so, their ideas are focusing on KCT and KCS, respectively.

Observation Guides

The observation guide did not always yield data related to MKT for two main reasons. First, during small group conversations participants were often hard to hear because they were whispering. Second, they often drew pictures instead of verbalizing their ideas. Two guides from the immersion professional development, however, did yield relevant data. These will be described in the paragraphs that follow.

School A–Immersion 2. In this session, the researcher was observing a group of two participants. They had been asked to decide which of three multiplication problems would facilitate students' thinking about the area model for multiplying fractions. The participants were given two minutes of individual, quiet time to examine the problems and then four minutes for partners to discuss the question posed. The following is a segment of that small group discussion.

Amber: (points to the aquarium problem) Kids will not make the connections on this one (talking about the floor plan).
Lindsay: (nods head in agreement)
PDL: (interrupting) What is wrong with the other ones?
Amber: This one is close but doesn't make them think about it. (pause)

Lindsay: I see a circle on that one.

PDL: Why is this so obvious? We were struggling last week to create these problems.

Amber: Cause you asked me to make things up.

PDL: Take a moment to draw the one you think it is.

Lindsay: I can't figure it out doing the area model.

Amber: I don't see kids thinking, "Oh area model!" They might think about it with this one but down here you have more seats in a row with this one but not necessarily.

In this small group exchange, the two participants grappled with which problem best depicted the area model. Specifically, they discussed the problems in terms of student misunderstandings and understandings, as seen in lines 1, 12, and 13. This type of small group interaction elicited a discussion focused on KCS.

School B–Immersion 2. During School B's second immersion session, the researcher was observing a group of two participants. The professional development leader had asked the participants to revisit a problem from the previous week, Measuring Scoops. They were told to represent and solve the problem with the given manipulatives and were given five to six minutes to work with their partner. The following is that group's interactions during those five to six minutes.

Jill: So this is a whole like she said.

Stacy: Wait, six made the whole so we need… (pause)

Jill: Six made the whole then three when broken in half (pause) but I can't understand what she had (giggling).

Stacy: So are we doing a whole and a whole and two and a half cups.
Jill: (counts them) One, two, three, four five, six… (pause)
Stacy: Right, if he had a measure of a third that would be a third and this would be a third.
Jill: This is a whole, and it takes three thirds to make a whole.
Stacy: I see what you're saying. Wait. Say it again.
Jill: This is a whole and it takes three thirds to make a whole.
Stacy: So this is a third, this is a third.
Jill: Right. So this is seven and one half cups. A part of it left over cause originally it is sixths so what is left is one out of sixths.
Jill: This is two cups but it asks for scoops?
Stacy: So this is one scoop and this is a little of another… (pause)
Jill: It's one sixth of a cup so it is one half of a scoop cause (pause) it was interesting. I saw it last week before we left. Oh, this was six and broken in half (pause) three out of six is (pause) one scoop is a third of a cup and this is two (pause) so six scoops and this would make a whole so seven scoops and this one third is one sixth but (pause) we said this was a half (pause) oh because this is two of these and a whole of this.
Stacy: That makes sense cause this is a whole but it is half of this so it is seven and half scoops.
Jill: But can we say it again? (giggling)

In an examination of this small group exchange, the two participants struggled with not only justifying their representation of the work but also examining how to interpret the remainder. Specifically, in lines 9 through 22, Jill tried to explain the process used in modeling the problem to Stacy. By focusing on modeling the division problem, this type of small group interaction demonstrated a discussion focused on SCK.

Research Question 1: What domains of Mathematical Knowledge for Teaching emerge in a professional development setting centered on the completion of mathematical tasks?

In reviewing the domains in the data set, one domain appeared the most often. At times, other domains were present in participants' conversations; however, the most prevalent domain was KCS. The session transcriptions, participant weekly reflections, and observation guides all revealed KCS emerging as participants from both schools engaged in the immersion experience.

Practice-based Professional Development

Mathematics education research has demonstrated that practice-based professional development supports the transformation many teachers of mathematics need (Smith, 2001). By providing teachers with opportunities to develop new mathematical knowledge through practice-based strategies, this model allows teachers to develop new techniques for instruction. Smith (2001) describes practice-based professional development as a program situated in practice. Specifically, the program should include the examination of materials taken from real classrooms. An example of this is a mathematical task accompanied by a set of student responses to that task (Smith, 2001). For this study, materials taken from classrooms included, but were not limited to, assessment items, textbook examples, student sample work, classroom vignettes and student videos. The next three sections contain the results yielded from the session transcriptions, participant weekly reflections, and observation guides from participants engaged in the practice-based professional development setting.

Session Transcriptions

School A–Practice-based 1. School A’s first practice-based professional development session focused on the models of division of fractions. Four vignettes will be shared to display the various MKT domains revealed during this session.

After reading a classroom vignette, the professional development leader allowed participants to discuss their initial ideas within small groups. At the end of approximately three minutes, the professional development leader asked the participants to share their initial thoughts with the whole group. The following vignette was taken from the sharing whole group process.

PDL: Okay, so let’s share just a little bit of our initial reactions to the vignette and then I have some specific questions that I will ask. Lizzy, will you share?

Lizzy: Well, (pause) lets see we (pause) ummm Marco got us all confused. I think my whole class, we talked about how the whole class would have been confused if they were trying to follow what he was trying to say.

PDL: Do you think it might help if they had all worked the problem themselves? I mean I didn't let y’all work the problem, I just said, “Here now lets get started.”

Lizzy: Right. If maybe Marco had had the opportunity to go to the board and explain what he did it may have made more sense. But I think they all got lost, my (pause) the rest of my classes would have gotten lost if they had had peaches and then tarts to put together plus thirds, two thirds and ten of something that would have just been too many things going on.

PDL: So you think the problem context itself would have (pause) might have pushed it?

Lizzy: I think we would have read it in sections.

In this vignette, a reflection of how the participants were struggling to make sense of the students' processes, based on a pictorial representation rather than an algorithm, was revealed. Lizzy's initial response to the classroom vignette centered on her own students. Lines 3 through 5 and 8 through 15 demonstrated her thoughts about how her students would be confused by listening to Marco's explanation. Additionally, in line 8 Lizzy described how students should "go to the board" and in line 15 how "we would read it in sections" which revealed her thoughts about classroom teaching. To this end, KCT was revealed during this section of the discussion.

Later during that same discussion, the professional development leader asked Lizzy what surprised her about the student responses in the vignette. Her response follows.

Lizzy- Well, again I would have had it drawn and I think we probably have more visual student learners than we had auditory because, and we're probably the reason, because all of us have to see it drawn as well so that's how we present it most of the time.

As Lizzy described the role of learning styles in instruction, her response was focused on her knowledge of her own students and her teaching. Ideas about her pedagogical knowledge were exposed, specifically KCS and KCT.

The professional development leader also asked Alicia the same question. The following is her response.

Alicia: I think my kids would have probably worked it out cause they, I don't know what their problem is with drawing because I draw pictures. I mean, not saying they would have gotten the correct answer but they would have tried to work it out.

PDL: Okay, so when you say they would have tried to work it out, what are you hypothesizing they would write down?

Alicia: I think they like (pause) we're not dividing fractions yet, so I don't know they would probably try to multiply two thirds times (pause) I dunno.

PDL: So you think they would be inclined to take the numbers and do something with them?

Alicia: Mine are bad about doing that. They see two numbers and just do some math with these numbers. They don't actually see what is going on.

In this vignette, Alicia predicted how students would respond to the task. Therefore, Alicia's response was rooted in her knowledge about her own students. Lines 29 through 30 demonstrated her theories about student misunderstandings regarding strategies for solving word problems. Therefore, her response was KCS in nature.

In the next vignette, Lindsay and Amber were asked to predict what equation students would write to match the problem within the classroom vignette. The following were their responses.

PDL: Okay so, my first question was if these students in this vignette, if their teacher now said write an equation or a number sentence to match this problem, what do we predict they would write. So, Lindsay, what were y'all saying related to that?

Lindsay: Okay, what do you think they would do?

PDL: Uh-huh, for an equation.

Lindsay: Okay, we were saying um thirty divided by, I mean thirty thirds divided by two thirds is equal to 15 over 1.

PDL: Okay, so what is it about the work that made you think that's what they would write?

Lindsay: Well, I guess, after kinda looking at what Kenny said you know he added up with the three thirds for each peach that he has and so that's kinda what we did. We added up each peach that we had and we came up with the thirty thirds.

PDL: Okay, so this is based on Kenny's?

Lindsay: No, wait, yeah, that's based on Kenny's operations and you know knowing that teaching the two thirds per tart so… (pause)

PDL: Amber, does that match the equation that you all were thinking?

Amber: Ummmmm, no, not exactly. Um, we weren't sure how they would write an equation based on what Kenny said.

PDL: Okay, but based on the other stuff?

Amber: Based on what Marco said I think they would do anything they could to avoid dealing with fractions, and they would look at that picture and say okay 10 peaches cut into three parts is ten times three which would give them thirty. And they'd say now I need two for each tart so then they would take their thirty and divide it by two to get their fifteen so that they had, they didn't have to deal with fractions.

In this vignette, Lindsay and Amber were asked to predict what equation students would write to match the problem within the classroom vignette the group had just read. In lines 51 through 56, Amber provided a mathematical process which she felt students might employ when solving the problem. In doing so, she referred to knowledge of students in general versus the previous segments where Alicia and Lizzy were asked a similar question but responded based on their knowledge of their own students. Each of the previous vignettes allowed a discussion focused around KCS to occur.

School A–Practice-based 2. The second practice-based session with School A focused on the algorithm for the division of fractions. Two vignettes will be shared to capture how various MKT domains were revealed during this session.

At the beginning of this session the professional development leader engaged the participants in recalling three things they knew about the repeated subtraction model for division. Following that conversation, the professional development leader provided the participants three student work samples from the Measuring Scoops task. The participants were given time to think about how the student work samples were similar to the student video watched the week before. Additionally, they were asked to relate the student work samples to the repeated subtraction model. A lengthy discussion in which the participants were grappling over the correct solution to the problem followed. Upon the resolution of that part of the question, Amber gave her response to the second part of the question, "How does [the Measuring Scoops video of the student working] relate to repeated subtraction?" The following is a segment between the professional development leader and Amber.

Amber: Yeah. Well, when I first (pause) when you look at it at first it's like ummm I'm almost thinking repeated addition not repeated subtraction because there's already a picture there. So I have to think about it like there's an amount of sugar and I'm actually taking a third scoop out so that's my actual subtraction so I'm trying to actually visualize the actual action rather then using the picture itself.

PDL: Okay, so it's a little bit more challenging to just look at this end product. Is that what you're saying?

Amber: To me it was.

PDL: You kinda have to go back and think about the context?

Amber: Cause I didn't have to do the splitting it up into the pieces. It was already there for me.

PDL: Okay, so in terms of what we wrote before you all are saying you subtract that same fraction out over and over. And what are we subtracting out each time?

Erin and Amber: That one third.

PDL: That one third. And we did it 'til we got to zero. Did we do that?

Erin: Well… (pause)

Amber: Well, until we got to one sixth.

PDL: Until we couldn't take anymore out. Okay. And we, did we use any common denominators?

Amber: Well, when we did one third, one third, and one third but when we got to what was left we didn't have the common denominator in that one.

PDL: Okay, but we didn't employ an algorithm either so probably if we had an algorithm that would come into play. And did we count the number of times you subtracted that out to get your answer?

Amber: Yes.

PDL: Okay, so very nice that our ideas from the beginning can help us think about how this is related to repeated subtraction. Very nice.

In this vignette, Amber and the professional development leader discussed how the video of the student working Measuring Scoops was related to the repeated subtraction model for dividing fractions. As this conversation unfolded, Amber focused her ideas on her own thinking processes for solving the problem. This conversation was mathematical in nature, hence, CCK and SCK were revealed at this point of the session. Later in the discussion, the participants were given two student sample responses to another problem. The following exchange of ideas in relation to interpreting the student responses ensued.

PDL: All right then, my next question is how does this stuff, cause that's a lot of stuff, help us or how does that relate to this over here? How are these similar? Erin, what was something you all pointed out about how that was similar to this?

Erin: Well, they realized that using the snap cubes they couldn't do the third so they did the sixth. They broke them into sixths like this student did, fifteen sixths and two sixths but they just had a row of six and a row of six and then a row of three cause they understood that a half would be a row of three.

PDL: So this group figured out they needed six because of that half three thing and then you saw the six over here being our common denominator. Nice. Lindsay, something from your group.

Lindsay: Okay, ummm (pause) well you know they ended up with, we just kinda went through step by step comparing how they (pause) ummmm took, I may need help, but you know the six cubes ended up being the common denominator then um the two cubes that was you know what they were gonna divide by and um they ended up with the fifteen altogether because they had to um have fifteen in order to pair them up at the end and um that’s where the two came in so the fifteen divided by two and the six was the common denominator and they ended up with the seven and half once they figured them up.

PDL: Okay, so y’all said in addition to what they said. So it sounds like what you’re saying is this idea that they have the six down here which we said that but that there's fifteen of those six and they are pairing them up and you saw the fifteen being in two rows. Okay.

In this section of the vignette, the participants navigated through the thought processes represented in the student sample work. The depth in which the participants were able to focus their ideas around the mathematics revealed SCK. Additionally, they were able to make sense of and reason through the student work, which again yielded SCK.

School B–Practice-based 1. For School B, the first practice-based session focused on the area model for the multiplication of fractions. Two vignettes will be shared in this section because they best depicted how various MKT domains were revealed during this session.

To start this session, the professional development leader asked the participants to decide which of four representations taken from a multiple-choice item correctly represented three times four using the area model, a previously explored topic. The following vignette is how that conversation unfolded.

PDL: Okay, so Jill, your card was pulled so tell us what y’all were saying in terms of these four problems that we had.

Jill: We were saying the one with (pause) the one with the grid because when you look at area you have the little squares. The little unit squares to represent area. And then there is three going down and four going across and then that gives you twelve.

PDL: Okay, is that what you all were saying?

The other group- (nod heads)

PDL: I'm sorry, Jill, I cut you off. Go ahead.

Jill: Oh no, that's it.

PDL: I did not mean to cut you off. Okay, but you all agree with that. Is that your reasoning? That we all were thinking, or is it something different in terms of why you selected that one? Carrie, will you kinda share what you all were saying.

Carrie: Well, we thought that every time you see area you usually see the box on the grid and third grade that's where they learn this so I also remember it from last year.

PDL: Okay, nice. So what model is this first one up at the top? Do y'all remember what that one would be called? (pause) Or the second one or the last one (pause)

Carrie: The second one is an array.

PDL: Okay, so we have an array. What are the other two?

ALL: Number line.

PDL: Number line, equal-sized jumps on the number line, sure.

Carrie: Ummm (pause) equal-sized groups?

PDL: Equal-sized groups on the first one. Nice. Okay, so we're seeing this area, and we're noticing that the biggest characteristics there are the squares in there. Those square units as Jill referred to. Cause that's how you measure area. Very nice.

In this vignette, Jill and Carrie shared their ideas about which of the four representations taken from a multiple-choice item correctly represented three times four using the area model. This warm-up activity engaged the participants in a discussion centered on the mathematics within the assessment item. Specifically, in lines 14 and 15 Carrie discussed how in third grade this content was learned. Therefore, KCC was revealed. Additionally, as Carrie responded to the professional development leader in lines 15 and 22, SCK was the dominant domain.

A continuation of this session included participants examining student work samples related to the Mary's Casserole task. In the next segment of the vignette, participants were discussing what about the student responses was interesting to them. Peggy, Jill, and Carrie are sharing their ideas in the segment below.

PDL: Okay. So what caught your attention about the green one? Was it the way it was drawn? Or the halves weren't quit right?

Peggy: First thing I looked for, did it divide it between parts?

PDL: So y'all were also looking for that.

Peggy: Uh-huh and then did it divide the thirds up?

Jill: And what the green one did different from the blue with the squares is he used the dotted lines to show where they went back and split and the thirds into halves versus the blue one is kinda hard you may think they started off with six instead of thirds.

PDL: Okay, so you're not clear what they drew first. I noticed the pink one has a different answer. Carrie, what were you all saying about that pink one?

Carrie: Ummmmm (pause) its kinda two things. We were saying that if they associated a literal and actual casserole and how you cut the square and maybe that's why they did that. Then when I kept looking at it, they did split it up into three parts and maybe they thought one half of one literal third. I mean I don't know where the fourth came in though. I'm not sure why they did that. It is four pieces if you count it all together.

Peggy: Right, I think that's how they got that the one fourth, they counted all the pieces.

PDL: Okay, so assuming that what they did, that they said oh there's four pieces and that’s one of them. What is it that that student is not understanding?

Jill: The equal parts.

PDL: The equal-sized parts, right. Yes, so we see that. Okay.

In this vignette, the participants were examining work samples from the Mary’s Casserole task and discussing interesting aspects of the student work. In doing so, this segment of the vignette yielded a conversation in which participants were engaged in describing students’ mathematical reasoning. From this specific segment of the conversation a deeper level of knowing was present. To this end, SCK dominated the conversation.

School B–Practice-based 2. The second practice-based session with School B was focused on the algorithm for the multiplication of fractions. Two vignettes will be shared in this section to demonstrate the MKT domains that were revealed during this session.

To start this professional development session, the leader recorded participants’ ideas about the multiplication of fractions on chart paper. After doing so, the leader provided the participants with three problems taken from a textbook. The participants were told to decide which one would help students think about the area model for multiplication. Upon narrowing down the choices to two, Peggy, Stacy, and Carrie provided the following thoughts.

PDL: Okay, so what did y’all decide is the big difference between the aquarium and the theatre problem?

Peggy: We were saying that the aquarium was one big space. And the theatre was little spaces together thinking the seats were little pieces put together.

PDL: So here would be my question, if you want your students to think about the area model which of these problems, we are titter-tottering back and forth, but which of these problems is going to clearly thinking about your students area? And what is it about that problem that is definitely going to get them thinking about that area model?

Stacy: Because, think about it now (pause) squares would each take up a square… (pause)

Carrie: I tried to say that but you shot me down (giggling).

PDL: Let me rephrase my question because that's a good point, maybe the little seats could be like the squares in area. If they're going to think about the area model, the area model says that in order to multiply two fractions you have a rectangle and those two fractions are the dimensions of that rectangle. So which of these problems is going to get those students to read it and draw a picture and those factors are the dimensions of the rectangle?

Carrie: Aquarium.

PDL: What is it about the aquarium?

Carrie: The rectangle.

PDL: Sure, okay, so what about the theatre is it that we are going we might not get a rectangle with that one?

Carrie: The theatre is circular (pause).

In this vignette, participants shared their ideas related to two textbook problems. As they struggled to reason through which one would help students think about the area model for multiplication, Peggy was limited to her own understanding of the mathematics, thus CCK. This was reflected in lines 3 through 4. In lines 13 through 18, the professional development leader reminded the participants of the stipulations of the area model. In doing so, Carrie was able to appropriately answer the question posed, hence SCK.

As the discussion continued, the professional development leader asked the participants to review sample student responses to the following sentence starter that was written on chart paper, “When you multiply fractions you multiply straight across because . . .” The following vignette is part of the discussion that unfolded.

PDL: So let’s share across the groups now. Which of the three is the one you read and it makes the most sense to you? Peggy?

Peggy: The last one.

PDL: Okay, and what is it about that one that makes sense to you?

Peggy: Well, um as soon as I read it, it caught me. The others I had to go back and read.

PDL: Then how is it different from the others?

Peggy: Ummmm well, I tell you actually when I read both of these the second and third pretty much say the same thing. I couldn't understand the first one. I said that it didn't mean that he or she didn't know what he was talking about they are just sayin’ it in a different way. One thing that got me was the subunits.

Stacy: Right!

PDL: Did y’all talk about that terminology?

Stacy: Right, but look at number two, look at all that good terminology we see the denominator, product, etc. When he said denominator I knew to go to the bottom numbers but when he said two times three, I had to go back up there and think two times three and I didn't read all that (giggles). I like number two.

PDL: So number two isn't necessarily using the numbers from the problem. Is that what you're saying? It is labeling them numerators?

Stacy: Right! So that led me to believe whatever numbers they whatever problem he did he would know what to do because he knew to do the denominator and the numerator.

PDL: Where this one is written specifically for this problem?

Stacy: Right.

PDL: What is it, what do you think this student is thinking up here about this subunits? What's going on with that?

Peggy: I’m thinking he's talking about each individual (pause) of the subunits because the last sentence says (pause) subunits by four subunits the whole thing is three subunits by four subunits. So he's trying to find the area of the entire unit instead of just the shaded area, that's what I think.

PDL: Why does he say subunits and not just units? Why doesn't he just say the whole thing is three units by four units?

Peggy: I dunno (pause) we would have to ask his teacher and see what he is thinking.

Carrie: He is saying the smaller pieces are subunits maybe I don't know I'm trying to give him the benefit of the doubt.

In this portion of the vignette, participants were discussing which student response to the sentence starter, “When you multiply fractions you multiply straight across because . . . ” made the most sense to them. In doing so, Stacy focused her ideas around how the student response she preferred was the one that used correct terminology. This was illustrated in lines 38 through 46.

Similarly, in lines 51 through 54 Peggy discussed how the student response she noticed might have misused the terminology of “subunits.” Both participants focused on hypothesizing about that student’s understanding of the mathematics, hence SCK is revealed at this point in the session.

Participant Weekly Reflections

School A–Practice-based 1. During this session, participants continued their discussion about the different representations for the division of fractions while moving towards a conversation about the division algorithm. The overall goal, therefore, was to have participants develop the algorithm for division of fractions. At the conclusion of the session, participants completed a participant weekly reflection. This specific reflection required participants to provide three things they noticed, two questions they had, and one big idea from the session. Through this reflection, participants were provided the opportunity to express openly their thoughts or concerns on which they were focusing as a result of the professional development. In the following paragraphs, three sample participant weekly reflections will be shared along with the MKT domains that were present in each reflection.

Exit Ticket

How do the Peach Tarts and Measuring Scoops problems support students' thinking about division with fractions?

They help me to see how students look at problems differently. Some use pictures & others use computation methods. Students have to be able to read & think about problems (see it visually) to know how to divide the parts.

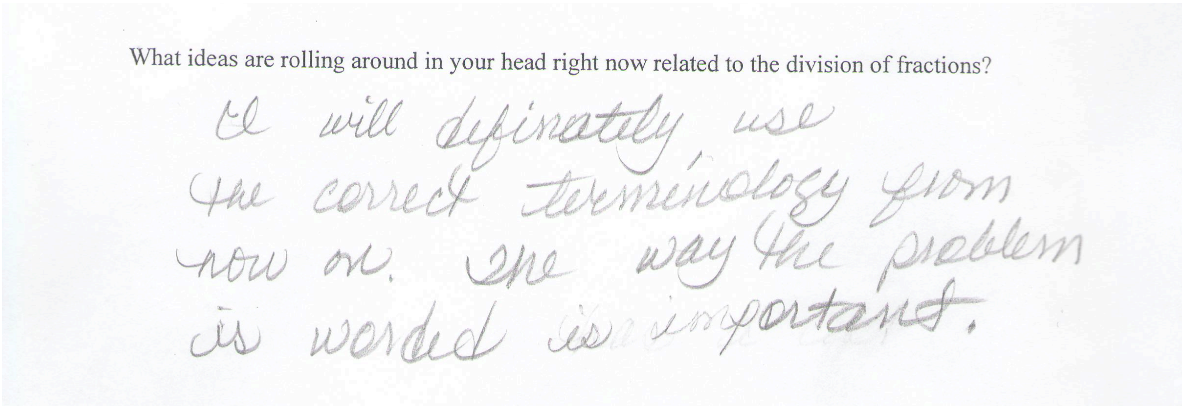
Figure 10a. One participant's weekly reflection.

How do the Peach Tarts and Measuring Scoops problems support students' thinking about division with fractions?

*Both problems help give the students a visual picture to work with. This allows them to see the seperation and dividing.

Figure 10b. One participant's weekly reflection.

In examining these two participants' reflections (see Figure 10) to the first part of the reflection, the researcher noted the responses were centered around the understandings students gain when the problem contexts facilitate creating representations. In Figure 10a, the participant focused her response on student thinking. Specifically, she related her thoughts to the types of strategies students employ when working division problems. Additionally, she mentioned a need for students to "see it visually." Similarly, in Figure 10b, another participant focused her response on the benefits of visualizations for students. Therefore, these participants' weekly reflections revealed KCS because they were thinking about how the students would best learn the content.


What ideas are rolling around in your head right now related to the division of fractions?

I will definately use
the correct terminology from
now on. The way the problem
is worded is important.

Figure 11. One participant's weekly reflection.

In examining the weekly reflection in Figure 11, the participant has focused on the use of correct terminology. The researcher noted the response was specifically centered around the participants' own use of correct terminology use in the classroom. Therefore, this participant's weekly reflection revealed KCT because she was focusing on the importance of teaching and using correct terminology.

School A–Practice-based 2. During this session, participants continued their discussion about the different representations for the division of fractions while moving toward a conversation about the algorithm. The overall goal, therefore, was to have participants develop the algorithm for division of fractions. At the conclusion of the session, participants completed a participant weekly reflection. This weekly reflection was designed to engage the participants in reflecting on three things they noticed, two questions they had, and one big idea from the session. In the following paragraph, one sample participant weekly reflection will be shared along with the MKT domains that were present.

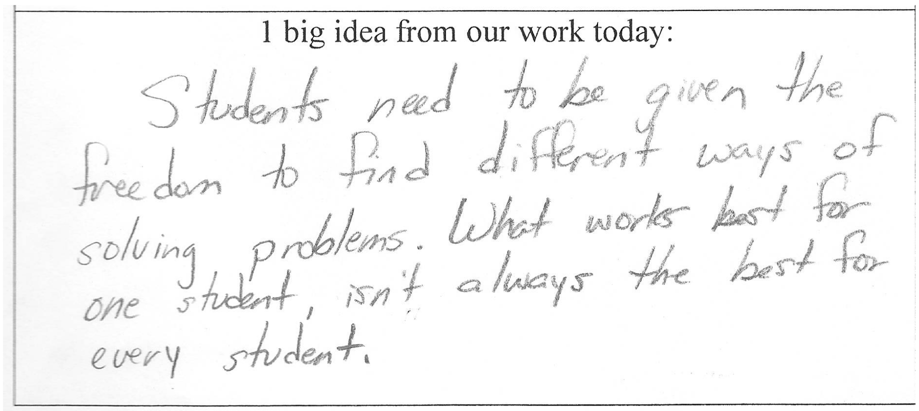

1 big idea from our work today:
Students need to be given the freedom to find different ways of solving problems. What works best for one student, isn't always the best for every student.

Figure 12. One participant's weekly reflection.

An examination of this response (see Figure 12) revealed that the participant was focused on the importance of multiple representations. Therefore, the researcher noted the participant's response was centered around KCS because she was thinking about what would be the best practice for students.

School B–Practice-based 1. During this session, participants examined different representations for multiplying fractions. The overall goal was to introduce participants to the area model for multiplying fractions recognizing this would support making sense of the algorithm. At the conclusion of the session, participants completed a participant weekly reflection that was designed to engage them in reflecting on the usefulness of the model as well as how they could utilize the information in the future. In the following paragraphs, three sample participant weekly reflections will be shared along with the MKT domains that were present in each reflection.

Think about the information we have discussed today. How will you use this information in the future?

When we finish talking about multiplication of
fraction I hope I can bring it back to the
classroom to help out in other areas. Today I
was confused about the area and multiplying
fractions

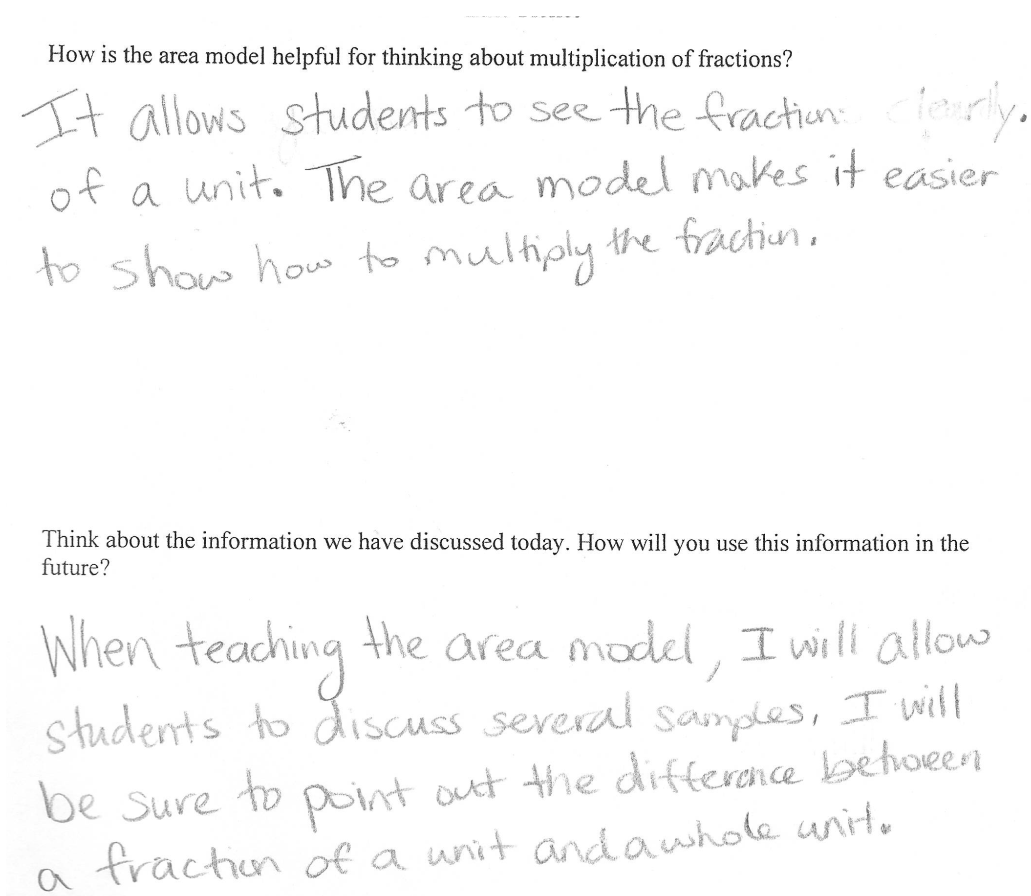
Figure 13a. One participant's weekly reflection.

Think about the information we have discussed today. How will you use this information in the future?

I will use this information to help those
students who still have a hard time
understanding that fractions are parts
of a whole. I really liked the reverse
problem (James Problem), so I will use
this with students as well.

Figure 13b. One participant's weekly reflection.

An examination of the reflections in Figure 13 revealed that participants focused on both their classroom practices and students. In 13a, the participant has described that she was still confused but hopeful that upon the completion of the professional development, she would be able to use the information in her classroom. In doing so, her ideas focused on KCT. The participant's response in Figure 13b focused on how the information will help students who are struggling with understanding fractions. In doing so, her ideas focused on KCS. In Figure 14, one participant shared similar ideas.



How is the area model helpful for thinking about multiplication of fractions?

It allows students to see the fraction clearly. of a unit. The area model makes it easier to show how to multiply the fraction.

Think about the information we have discussed today. How will you use this information in the future?

When teaching the area model, I will allow students to discuss several samples, I will be sure to point out the difference between a fraction of a unit and a whole unit.

Figure 14. One participant's weekly reflection.

An examination of this reflection revealed that the participant focused her ideas around students in the first part and teaching in the second part. Specifically, she has described how students can utilize the area model for multiplication and how she will point out the difference between fractions and wholes. In doing so, her ideas focused on KCS and KCT, respectively.

School B–Practice-based 2. During this session, participants continued their examination of the models for multiplying fractions while moving towards an understanding of the algorithm. At the conclusion of the session, participants completed a participant weekly reflection. This first question on the reflection was designed to engage the participants in reflecting on observations made about the mathematical task they were examining during the session. In addition, the second question on the reflection was designed to reflect on ideas about the area model and multiplication of fractions. In the following paragraphs, one sample participant weekly reflection will be shared along with the MKT domains that were present.

Reflect back on the problem you and your partner created last week. Now that we have looked at the textbook problems, write three or four observations that you can make about your problem.

I think that the students this week understands
the process of multiplying fraction. Last week
I was a little confused but today it is clear.
The problems we had today made sense even
with the subunits.

What ideas are rolling around in your head right now related to the area model or the multiplication of fractions?

I think we I get to multiplying fractions
it will make me full more confident. It
will make me want to try different
things with the students. The problem
from a last is clear to me now.

Figure 15. One participant’s weekly reflection.

An examination of this reflection in Figure 15 revealed that the participant focused on both student thinking and her own disposition towards the multiplication of fractions. In her first response, she described how the student was thinking about the targeted mathematics. In doing so, her ideas focused on KCS. In addition, an examination of the second response revealed how the participant reflected on her own disposition towards multiplying fractions and how this will impact her instructional decisions. Therefore, her ideas focused on the KCT domain.

Observation Guides

The observation guide did not always yield data related to MKT for two reasons. First, during small group conversations participants were often hard to hear because they were whispering. Second, they often drew pictures instead of verbalizing their ideas. One guide from the practice-based professional development, however, did yield relevant data. This data will be described in the paragraph that follows.

School A–Practice-based 1. In this session, the researcher was observing two participants in their small group. They were discussing the Measuring Scoops student video. The professional development leader had specifically asked the groups to discuss what was interesting to them about the student's answer in the video. The following is their discussion.

Erin: Yeah (pause) I really liked the way she did it… (pause)
Lizzy: How she broke it down (pause) there isn't any repeated subtraction (pause)
all right what's interesting about her answer?
Erin: She knew the triangle was half.
Lizzy: Yeah.
Erin: It was related to the shapes you know… (pause)
Lizzy: (writing on her paper while Erin watches and then whispering)

Lizzy: Did it work?

Erin: Yeah, but for fractions you need common denominators but what would throw them off is half of a third which… (pause)

Lizzy: But if they got common denominators… (pause)

Erin: If they do this they would do seven or seven and a sixths but no, if they get seven and a sixths of a third… (pause)

PDL: How did the student know it was half?

Erin: How did she know it was half of the cup? When she is using the pattern blocks she is seeing it as one whole cup so the green triangle is one half of the blue scoop so she isn't seeing it as a fraction, she is seeing it as half of the piece.

In an examination of this small group exchange, the two participants grappled with how the student in the video modeled and solved the Measuring Scoops task. In lines 1 through 4, Erin and Lizzy tried to explain the process the student in the video used. In doing so, they engaged in a conversation focused on SCK. As the conversation continued, Erin continued to focus on the mathematical understanding of the student in the video. This is reflected in lines 15 through 17 as she attempted to explain the student's thinking process in a more specialized way, hence SCK.

Research Question 2: What domains of Mathematical Knowledge for Teaching emerge in a professional development centered on the analysis of student work?

In reviewing the domains represented in the data set, the domain that appeared more often than the rest was SCK. At times, other MKT domains were present in participants' conversations; however, SCK was the most prevalent. The session transcriptions and participant weekly reflections revealed SCK as participants from both schools engaged in practice-based professional development setting.

Summary

This chapter presented all qualitative results of data obtained as two schools engaged in both immersion and practice-based professional development sessions. A synopsis of results from each tool was provided to give the reader a vision of the type of MKT domains that emerged during each professional development setting. Although the excerpts presented are limited in scope, they reflect the focus of the participants' thought processes as they engaged in the professional development. In the next chapter, the researcher will provide discussion, implications, and recommendations around the data.

V. DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

Introduction

The purpose of this qualitative research study was to examine how different foci of professional development revealed different domains of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008). In one school setting, the professional development focused on engaging teachers in discourse surrounding teachers' completion of mathematical tasks, namely the immersion experience. In the second school setting, the professional development focused on engaging teachers in discourse surrounding student work associated with the mathematical tasks, namely practice-based professional development. In an effort to capture instances of Mathematical Knowledge for Teaching being revealed in those two different settings, the researcher posed the following questions.

1. What domains of Mathematical Knowledge for Teaching emerge in a professional development setting centered on the completion of mathematical tasks?
2. What domains of Mathematical Knowledge for Teaching emerge in a professional development centered on the analysis of student work?

This chapter begins with a discussion of the results to the two research questions. The section includes discussion points taken from the results as well. Finally, implications and recommendations are provided.

Discussion

In an attempt to respond to each research question, the researcher utilized qualitative means for analyzing data taken from video recorded sessions, participant weekly reflections, and observation guides. Transcriptions from the recorded sessions and the participant weekly reflections provided the researcher with the bulk of her findings.

In reviewing the domains represented from the immersion setting data, the domain that appeared more often than the rest was KCS. In reviewing the domains represented from the practice-based professional development data, the domain that appeared more often than the rest was SCK. At times, other MKT domains were present in participants' conversations and written work; however, KCS and SCK were the most prevalent. The following paragraphs capture the discussion points that stemmed from those findings.

Responding to Questions in the Immersion Experience

During the immersion experience, the researcher often identified instances in which the participants did not directly answer the professional development leader's questions. These questions were mathematical in nature, often expecting the participant to provide a mathematical justification. The participants' responses, however, were focused on the KCS domain. This was represented in two ways. First, when the participants were struggling with an idea themselves, they often answered questions with responses that reflected how their students would lack the ability to do certain tasks. In other words, the participants seemed to think that if they could not understand the mathematics embedded in the tasks, the tasks were not appropriate for students as they, too, would be unsuccessful. Second, when the participants were faced with a challenging mathematical question they would often respond with, "My students would like that model." For example, if participants were looking at the representations generated by the group, they justified their choice of representation based on what students like rather than provide a mathematical justification. Therefore, when the participants were engaged in the immersion experience, they often eliminated the mathematics from their reasoning altogether.

Responding to Questions in Practice-based Professional Development

The way in which participants responded to questions in the immersion experience was in contrast to how they responded in the practice-based professional development. In the practice-based setting, which naturally lent itself to the use of student work samples, participants worked to make sense of the mathematics represented within the student work. In doing so, the participants' responses were similar to that of the participants described by Groth and Burgess (2009). In their study, these researchers found that classroom artifacts, namely student work samples, motivated the teachers to talk about the mathematics content. By contrast, however, the teachers in the Groth and Burgess study also talked about the students and about the pedagogical content knowledge needed in their instructional practices (Groth & Burgess, 2009). Such was not the case in the current study where the participants focused solely on the mathematics. Still, Kazemi and Franke (2003) stated that, "Making sense of student's strategies could be an indirect way for teachers to wrestle with the mathematical ideas themselves" (p. 7). To this end, this qualitative study compliments, and in some respects enhances, past research.

Coding the Domains of MKT

The researcher's choice to use the concepts of multiplication and division of fractions during the professional development possibly limited the type of domains that may have otherwise emerged during the sessions. The participants' responses to the professional development leader's questions and interpretations of students' mathematical ideas were often flawed mathematically. Their lack of knowledge about the concepts with which they were engaged hindered the researcher during the data analysis process from finding more instances of

MKT being revealed. Therefore, the researcher had to shift the lens of analysis which she was using initially.

In phase one of data analysis, the researcher was looking for instances of participants' *having* MKT. This limited her findings because, as previously stated, the knowledge of the multiplication and division of fractions that the participants brought into the professional development settings was tenuous, at best. The tasks utilized in the professional development settings served to expose their lack of a knowledge base, as opposed to enhancing or deepening it. This realization led the researcher to consult her advisor and discuss the results. In doing so, the advisor recommended the researcher examine the data with the lens of "on what domains of MKT were the participants focusing their discussions and reflections?" Thus, the researcher had to shift her thinking from focusing on the participants' mathematical accuracy toward the types of ideas represented in their conversations and written work. When the researcher coded a part of a session transcription as SCK, for example, that coding was not an indication that the participant possessed SCK but rather an instance in which that participant was articulating ideas related to SCK. Therefore, the results of this study are not meant to indicate that either professional development setting promoted growth in MKT. Rather, the results indicate that as teachers engage in the two different settings their interests lie in different domains of the MKT model.

Levels of SCK

Once the researcher shifted her lens, she continued to struggle with the coding process. Specifically, the researcher often struggled with coding a discussion or reflection with CCK or SCK. The researcher sought the advice of her advisor in an attempt to resolve this issue. A debate about the use of terminology was the turning point for the researcher. For example, when coding the participants' recognition of the repeated subtraction model, initially the researcher coded the instance as CCK. Upon discussing this issue with her advisor and consulting the original MKT work of Ball and colleagues (2008), the researcher coded the instance as SCK. Again, the researcher was not suggesting that the participants had SCK but rather their conversation was focused on a level of SCK. This, in turn, leads the researcher to suggest the possibility that there are various levels of knowledge within the SCK domain.

Implications and Recommendations

As previously stated, the researcher struggled to code instances of the phenomenon as either CCK or SCK. As Hill, Schilling, and Ball (2004) found in their work, mapping the levels of MKT precisely has yet to be tackled. Based on this study, there appears to be a need to examine the depth of knowledge that exists within SCK. To this end, the researcher recommends that mathematics educators focus their attention toward developing levels within SCK.

In the mathematics education research community, attention has shifted away from CCK with more attention given to other domains of the MKT model (Hill, Schilling, & Ball, 2004). The researcher's literature review provided evidence that CCK is often excluded from research examining MKT altogether. Yet from this study, CCK appeared to play a relevant role in SCK. This notion aligns with Hill, Schilling, and Ball's ideas that "CCK is related to SCK though not equivalent." Therefore, there is a need for researchers to examine the role of strong or weak CCK in the development of MKT.

Additionally, the researcher recognizes that this study's limitations require that more work is needed to verify results. Therefore, the researcher recommends that work, similar in nature, be done. When the results are verified, those who plan professional development will be better informed on the best practices for enhancing teachers' MKT.

In closing, this study revealed one last implication for those who plan professional development. If the primary goal of the professional development is to develop teachers' content knowledge, then the best means for motivating teachers to think within a particular subject matter domain would be the practice-based setting. Similarly, if the primary goal of the professional development is to develop teachers' knowledge of students, then the best means for motivating them to think within a particular pedagogical domain would be the immersion experience. The researcher, however, suggests perhaps a combination of the two settings. In doing so, the professional development could potentially be the best way for enhancing teachers' Mathematical Knowledge for Teaching.

List of References

REFERENCES

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14–46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59,* 389–407.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Reston, VA: National Council of Teachers of Mathematics.

Cohen, D. K., & Hill, H. C. (2001). *Learning policy: When state education reform works.* New Haven, CT: Yale University Press.

Cohen, S. (2004). *Teachers' professional development and the elementary mathematics classroom: Bringing understandings to light.* Mahwah, NJ: Lawrence Erlbaum Associates.

Common Core State Standards Initiative (CCSSI) (2010). *Common core state standards for mathematics. Common core state standards (College- and career-readiness standards and K–12 standards in english language arts and math).* Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved March 4, 2011, from http://www.corestandards.org.

Cramer, K. & Henry, A. (2002). Using manipulative models to build number sense for addition and fractions. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions* (pp. 41– 48). Reston, VA: National Council of Teachers of Mathematics.

Crespo, S., & Featherstone, H. (2006). Teacher learning in mathematics teacher groups: One math problem at a time. In K. Lynch-Davis, R. L. Rider, & D. R. Thompson (Eds.). *The work of mathematics teacher educators: Continuing the conversation* (pp. 97–115). San Diego, CA: Association of Mathematics Teachers Educators.

Groth, R. E., & Burgess, C. A. (2009). An exploration of two online approaches to mathematics teacher education. *Journal of Computers in Mathematics and Science Teaching, 28*, 221–246.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education, 35*, 330-351.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-155). Charlotte, NC: Information Age Publishing.

Huinker, D. (2002). Examining dimensions of fraction operation sense. In B. Litwiller & G. Bright (Eds). *Making sense of fractions, ratios, and proportions* (pp. 72–78). Reston, VA: National Council of Teachers of Mathematics.

Jacobs, V. R., Lamb, L. C., & Philipp, R. A. (2009). Professional noticing of students' mathematical thinking. *Journal for Research in Mathematics Education, 41*, 169–202.

Kazemi, E., Elliot, R., Lesseig, K., Mumme, J., Carroll, C., & Peterson-Kelley, M. (2009). Doing mathematics in professional development to build specialized content knowledge for teaching. In D. S. Mewborn & H. S. Lee (Eds.), *Scholarly practices and inquiry in the preparation of mathematics teachers* (pp. 171–186). San Diego, CA: Association of Mathematics Teachers Educators.

Kazemi, E., & Franke, M. L. (2003). *Using student work to support professional development in elementary mathematics* (Document W-03-1). University of Washington: Center for the Study of Teaching and Policy.

Kamii, C., & Warrington, M. A. (1999). Teaching fractions: Fostering children's own reasoning. In L.V. Stiff (Ed.), *Developing Mathematical Reasoning in Grades K-12* (pp. 82–92). Reston, VA: National Council of Teachers of Mathematics.

Loucks-Horsley, S., Hewson, P. W., Love, N., & Mundy, S., Stiles, K. E. (1998). *Designing professional development for teachers of science and mathematics.* Thousand Oaks, CA: Corwin Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

McDuffie, A. R., & Slavit, D. (2003). Utilizing online discussion to support reflection and challenge beliefs in elementary mathematics methods classrooms. *Contemporary Issues in Technology and Teacher Education, 2*, 446–466.

Mississippi Department of Education (2007). *2007 Mississippi mathematics framework revised*. Jackson, MS: Author.

Mississippi Department of Education (2010). *Mississippi accountability and assessment reporting system.* Retrieved July 23, 2010, from http://orshome.mde.k12.ms.us/ors/

National Center for Education Statistics (2009). *The nation's report card: Mathematics 2009* (NCES 2010–451). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council (NRC). (2001). *Adding it up: Helping students learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., et al. (2007). Effects of early field experiences on the mathematical content knowledge of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38,* 438–476.

Rasmussen, C., Yackel, E., & King, K. (2003). Social and sociomathematical norms in the mathematics classroom. In H. L. Schoen. (Ed.), *Teaching mathematics through problem-solving: Grades 6-12* (pp. 143–154). Reston, VA: National Council of Teachers of Mathematics.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Smith, J. P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller & G. Bright (Eds), *Making sense of fractions, ratios, and proportions* (pp. 3–17). Reston, VA: National Council of Teachers of Mathematics.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313–340.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–224). Charlotte, NC: Information Age Publishing.

Suzuka, K., Sleep, L., Ball, D. L., Bass, H., Lewis, J. M., & Thames, M. H. (2009). Designing and using tasks to teach Mathematical Knowledge for Teaching. In D.S. Mewborn & H. S. Lee (Eds.), *Scholarly practices and inquiry in the preparation of mathematics teachers* (pp. 7–23). San Diego, CA: Association of Mathematics Teachers Educators.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Whitcomb, J., Borko, H., & Liston, D. (2009). Growing talent: Promising professional development models and practices. *Journal of Teacher Education, 60*, 207–212.

Wiersma, W. (1995). *Research methods in education: An introduction* (6th ed.). Boston: Allyn and Bacon.

List of Appendices

APPENDIX A

Participant Weekly Reflection

Exit Ticket

How is the area model helpful for thinking about multiplication of fractions?

Think about the information we have discussed today. How will you use this information in the future?

APPENDIX B

Observation Guide

Handouts:	Materials:
• Attendance Sheet • Mary's Casserole problem • Manipulative handout • Book Excerpt	• Timer • Chart paper & markers

Mathematical Topic: Rational Number Computations

Daily Goal:

- **To link multiplication of fractions to the area model for multiplication.**

Mary's Casserole Problem
Distribute copies of the "Mary's Casserole" problem. Allow 3-5 minutes for participants to solve. Have participants utilize pictures and number sentences to represent the problem.

At the end of 5 minutes select various representations of the problem to be recorded at the board. Have small groups compare and contrast the representations, additionally 3 minutes.

Share observations and record on board. After all observations have been shared, focus candidates' attention on the representation that demonstrates the use of the area model for multiplication. Have participants think-pair-share: How is this representation related to the area model for multiplication? (3-5 minutes) Share out responses. Take any questions.

Now have participants use the area model to model and solve . Select one or two groups to share.

Summary (10 minutes)
Think-pair-share: How has our work with fraction computation today been different from what you typically do in your own classroom?

Have participants individually respond to the prompt in writing as well.

Field Notes

APPENDIX C

Qualifications of the Expert

The expert who led all professional development sessions was an Associate Professor of Mathematics Education at a university within the state. This expert had 14 years of experience working with pre- and in-service teachers. Additionally, this individual had 18 years of experience working with students. This expert had written and published numerous manuscripts about the teaching and learning of mathematics and standards-based instruction. The expert had presented personal research results at both the state and national level at such conferences as Association of Mathematics Teacher Educators (AMTE) and the National Council of Teachers of Mathematics (NCTM). Furthermore, she held the positions of president of a state affiliate of AMTE and of board member of a state affiliate of NCTM. To date, she has secured more than $1.5-million in grant funds in an effort to enhance professional development in the expert's state. Therefore, this individual's extensive experiences with teachers, students, teacher candidates, organizations, and grant writing indicated that she was highly qualified to lead the professional development sessions for this research study.

APPENDIX D

Information about a Research Study

Title: An Examination of Mathematical Knowledge for Teaching in Professional Development

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Description
We want to know how activities completed in professional development sessions help you to better understand the mathematics that you teach. To find out, we are asking that you participate in professional development by sharing your ideas about mathematics. All sessions will be video recorded to help us gather this information. We will explain the study to you and you can ask any questions you have about it.

Risks and Benefits
You may feel uncomfortable because you may not have thought about mathematics this way. We do not think that there are any other risks.

Cost and Payments
There are no costs, other than your time, for helping us with this study. In return for your participation, you will receive a problem-solving kit, a book about writing in mathematics, and continuing education units (CEUs).

Confidentiality
Your name will be removed from all transcripts of the professional development sessions and a code name will be assigned. Only the researchers will have access to the videos. All videotapes will be destroyed after being transcribed, approximately 6 months after the completion of the study. Any written work that is collected will be anonymous. Therefore, we do not believe that you can be identified.

Right to Withdraw
You do not have to take part in this study. If you start the study and decide that you do not want to finish, all you have to do is to tell Dr. Barlow or Ms. Harmon in person, by letter, or by telephone at the Department of Curriculum and Instruction, 310 Guyton Hall, The University of Mississippi, University MS 38677, or 915-1276. Whether or not you choose to participate or to withdraw will not affect your standing with the Department of Curriculum and Instruction, or with the University, and it will not cause you to lose any benefits to which you are entitled.

IRB Approval
This study has been reviewed by The University of Mississippi's Institutional Review Board (IRB). The IRB has determined that this study fulfills the human research subject protections obligations required by state and federal law and University policies. If you have any questions, concerns, or reports regarding your rights as a participant of research, please contact the IRB at (662) 915-7482.

APPENDIX E

Area Model (SCHOOL A)

Handouts:	**Materials:**
• Warm-up • Mary's Casserole • James' Problem • Exit Ticket	• Video Camera w/ tripod • Index Cards • Door prize (Travel Mug) • Chart paper & markers • Blue tape • Curriculum Frameworks • Food & drinks, ice in a cooler • Plain white paper

Goals:

- To model multiplication of fractions with the area model for multiplication.

Warm-up

Distribute copies of the warm-up problem. Allow 1 minute to think individually. Then, share in their groups' their responses. Share out whole group.

Mary's Casserole Problem

Distribute copies of the "Mary's Casserole" problem and a piece of plane white paper and marker. Ask participants to solve by drawing a large picture on the white paper with a marker and then writing a number sentence that represents the problem. Post the different representations of the problem at the board. After all pictures have been shared, focus participants' attention on the representation that demonstrates the use of the area model for multiplication. Think-pair-share: How is this representation related to the area model for multiplication?

Ask participants as a group to use the area model to model and solve $\frac{2}{3} \cdot \frac{4}{5}$. Again, distribute white paper so that they can record and share their representations. Swap and compare only if there are differences observed.

James' Problem

Distribute copies of James' Problem. Review the question. This task is to be completed collaboratively within the groups. Allow 5 – 6 minutes to work. As groups create their word problems, they should record them on chart paper. After time is called, ask groups to swap problems and compare/contrast the problems.

Share out. Possible debriefing questions include:

- How does your word problem help students to think about the area model for multiplying fractions?
- Possibly: Let's return to our four word problems for multiplication from a previous session. Is the problem you created today more like the movie theater problem or the checkerboard problem? Why?

Summary

Look at the framework. Think for a moment about how the work we've done today, is developed in your grade level as well as others. Now, talk about this in your group. Share out.

Exit Ticket

APPENDIX F

Area Model (SCHOOL B)

Handouts:	**Materials:**
• Warm-up • Mary's Casserole • James' Problem • Exit Ticket	• Video Camera w/ tripod • Index Cards • Door prize (Travel Mug) • Chart paper & markers • Blue tape • Curriculum Frameworks • Food & drinks, ice in a cooler • Plain white paper

Goals:

- To model multiplication of fractions with the area model for multiplication.

Warm-up

Distribute copies of the warm-up problem. Allow 1 minute to think individually. Then, share in their groups' their responses. Share out whole group.

Mary's Casserole Problem

Distribute copies of the "Mary's Casserole." As participants read the problem, post 4 different representations of the problem at the board. Have participants discuss each within their small group. After all pictures have been shared, focus participants' attention on the representation that demonstrates the use of the area model for multiplication. Think-pair-share: How is this representation related to the area model for multiplication?

Ask participants to examine the student work samples for $\frac{2}{3} \cdot \frac{4}{5}$. Assign each group one work sample to evaluate. Ask participants to check to see if the work accurately uses the area model to represent the problem. Allow 2 – 3 minutes to decide. Share out.

James' Problem

Distribute copies of James' Problem. Review the question. Distribute one chart paper with sample work to each group. Their task is to check the word problem to see if it aligns with the area model for multiplication. Allow 5 – 6 minutes to work. During this time, groups are to discuss the work and record on post-its three points of discussion related to the work to be shared with the whole group. After time is called, each group will present their work along with their points of discussion.

Share out. Possible debriefing questions include:

- How does your word problem help students to think about the area model for multiplying fractions?
- Possibly: Let's return to our four word problems for multiplication from a previous session. Is the problem you created today more like the movie theater problem or the checkerboard problem? Why?

Summary

Look at the framework. Think for a moment about how the work we've done today, is developed in your grade level as well as others. Now, talk about this in your group. Share out.

Exit Ticket

APPENDIX G

Overview of 4 weeks of Professional Development

Week	Activity	Immersion	Practice-based
1	Multiplication of Fractions: Models	A	B
2	Multiplication of Fractions: Algorithms	A	B
3	Division of Fractions: Models	B	A
4	Division of Fractions: Algorithms	B	A

VITA

Shannon Elizabeth Harmon was born to James and Peggy Harmon in Tupelo, Mississippi, in 1978. After graduating from Tupelo High School in 1996, Shannon attended Itawamba Community College and Mississippi State University. In May 2000, Shannon completed her Bachelor of Arts in Communications from Mississippi State University.

After obtaining her teaching license in 2003, Shannon taught special education students in grades 4 through 7 for four years. In June 2007, Shannon joined the Center for Mathematics and Science Education to work as a full-time masters student. In 2008, Shannon earned her Master of Education from The University of Mississippi. That same year, she joined the Project DELTA2 team as the graduate research assistant and continued her education as a doctoral student.

While attending The University of Mississippi, Shannon was active in teaching, research, and service. In the area of teaching, she worked for the Department of Curriculum and Instruction in The School of Education as a member of the methods in mathematics instructional team. Shannon also taught mathematics lessons in elementary classrooms while groups of teachers observed the instruction. In the area of research, Shannon co-authored four manuscripts, having two in press at the time of her graduation. Furthermore, she presented at both state and national conferences. In the area of service, Shannon served as the Member-at-Large for a state association. In 2010, Shannon received the Lamar Memorial Scholarship and was named the Outstanding Doctoral Student of the Year in Elementary Education.